Applied Soft Computing xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

A novel soft rough set: Soft rough hemirings and corresponding multicriteria group decision making

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ARTICLE INFO

Article history:
Received 4 December 2015
Received in revised form 22 August 2016
Accepted 23 August 2016
Available online xxx

MSC:
16Y60
13E05
03G25

Keywords:
Rough set
Soft rough set
MSR-hemiring
Decision making

ABSTRACT

In this paper, we investigate the relationships among rough sets, soft sets and hemirings. The concept of soft rough hemirings is introduced, which is an extended notion of a rough hemiring. It is pointed out that in this paper, we first apply soft rough sets to algebraic structure-hemirings. Further, we first put forward the concepts of *C*-soft sets and *CC*-soft sets, which provide a new research idea for soft rough algebraic research. Moreover, we study roughness in hemirings with respect to *MSR*-approximation spaces. Some new soft rough operations over hemirings are explored. In particular, lower and upper *MSR*-hemirings (*k*-ideal and *h*-ideal) are investigated. Finally, we put forth an approach for multicriteria group decision making problem based on modified soft rough sets and offer an actual example.

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1. Introduction

Rough set theory, was first proposed by Pawlak [35,36], has become an important tool for uncertainty management in a wide range of applications related to intelligent decision making systems, cognitive science, patter recognition, machine learning, image processing, signal analysis and many other fields [11,22,37–39,53,54]. According to the definition of rough sets, any subset of a universe can be characterized by equivalence relations. The equivalence relations in Pawlak rough sets are too restrictive for theoretical and practical aspects, however, so many investigators have generalized the concept of Pawlak rough sets by using non-equivalence relations. Some generalized rough set models can be found in [3,10,28,45,56,57]. On the other hand, rough set theory could be applied to algebraic structures [6,8,12,13,33,42,43,55].

With the development of uncertainty theory, Molodtsov [34] put forward a novel concept called a soft set as a new mathematical tool for dealing with uncertainties. After that, a wide applications of soft sets have been studied in many different fields including game theory, probability theory, smoothness of functions, operation researches, Riemann integrations and measurement theory [31,34]. In recent years, research works on soft sets are very energetic and progressing quickly [1,4,5,9,24,40,50]. In particular, Li [26] studied soft covering and its parameter reduction. Li [27] investigated the relationships among soft sets, soft rough sets and topologies. In 2010, Feng et al. [17] proposed rough soft sets by combining Pawlak rough sets and soft sets, rough soft sets can be regarded as a collection of rough sets sharing a common Pawlak approximation space. Recently, based on the idea in [17], Zhan [49] firstly applied rough soft sets to hemirings, and described some characterizations of rough soft hemirings.

On the other hand, hemirings (semirings with zero and commutative addition) provide an algebraic framework for modeling and investigating the key factors in different areas of mathematics as functional analysis, graph theory, formal language theory and parallel computation systems. Over these years, from both theoretical and practical needs, many authors studied the nature of (semirings) hemirings [19,20]. We know that ideals of semirings play a central role in the structure theory and are useful for many purposes. Many results in rings apparently have no analogues in hemirings using only ideals. In order to overcome this insufficient, Henriksen [21] defined the *k*-ideals. Afterwards a still more restricted, but very important, a class of ideals, called an *h*-ideal, has been

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http://dx.doi.org/10.1016/j.asoc.2016.09.012
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given and investigated by Izuka [23] and La Torre [44]. Furthermore, the relationships among rough sets, fuzzy sets, soft sets and semirings (hemirings) have been considered by many scientists [14,25,29,30,46,48,51].

It is worth noting that decision making in an imprecise environment has been showing more and more important role in real life. Researches on the applications of fuzzy set theory, rough set theory, soft set theory, as well as their hybrid models in decision making have attracted researchers' widespread interest. Recently, Zhang [52] introduced a method for multi-attribute decision making applying soft rough sets. Therefore, the another aim of this paper is to explore the applications of soft rough sets in decision making.

Soft set theory is a new approach to deal with uncertainties, and it focuses on the parameterization, while rough set theory as anther tool to deal with uncertainties, it places emphasis on granular. However, the real-world problems that under consideration are often very complex, so the roughness and parameterization may appear an actual problem simultaneously. Consequently, it is very hard to deal with those complex problems only by a single mathematical tool. Rough set theory and soft set theory as two different tools to deal with uncertainties, the most obvious is that there is no direct connection between these two theories. However, efforts have been made to establish some kind of linkage [7,18]. The most important criticism on rough set theory is that it lacks parameterization tools [31]. In order to make parameterization tools available in rough sets, the most important step is that Feng [17] introduced the concept of soft rough sets, where instead of equivalence classes parameterized subsets of a set served the purpose to finding lower and upper approximations of a subset. However, the soft set must be a full soft set in order to resolve theoretical and practical aspects. Recently, Shabir [41] pointed out that an upper approximation of a non-empty set may be empty and upper approximation of a subset X may not contain the set X on the soft rough sets which Feng introduced in [18]. To resolve this problem, Shabir modified this concept and proposed a class of revised soft rough set, which is called an *MSR*-set. The *MSR*-sets are not only no restrictions on the soft sets but also the underlying concepts are very similar to classical rough sets. As a result, the combination of the two aspects will be more effective when dealing with uncertain problems. Recently, Zhang [52] introduced a method for multi-attribute decision making applying soft rough sets.

Based on the above reason, it is an interesting work to discuss further on this topic. The present paper aims at providing a framework to combine rough sets, soft sets and hemirings all together, which proposes soft rough hemirings and studies the characterizations of soft rough sets in hemirings. Apply the soft rough sets to the hemrings, on one hand we can find the impacts of soft rough sets on hemirings. On the other hand, it also reflects the characterizations and properties of soft rough sets in hemiring structure. The combining of rough sets, soft sets and hemirings all together is not studied until now. And, in this article, we firstly apply soft rough sets to hemirings. This paper is organized as follows: In Section 2, we recall some concepts and results on hemirings, rough sets and soft sets. In Section 3, we propose some operations of modified soft rough sets. In Section 4, we investigate some characterizations of soft rough hemirings including their new operations. Finally, we put forth an approach for decision making problem based on soft rough sets in Section 5.

2. Preliminaries

In this section, we will review some basic notions about hemirings, soft sets and rough sets.

By a zero of a semiring $(S, +, \cdot)$, we mean an element $0 \in S$ such that $0 \cdot x = x \cdot 0 = 0$ and $0 + x = x + 0 = x$ for all $x \in S$. A semiring with zero and a commutative semigroup $(S, +)$ is called a hemiring. By one unit 1 on S, we means that $1 \cdot x = x \cdot 1 = x$ for all $x \in S$. For the sake of simplicity, we shall write ab for $a \cdot b (a, b \in S)$. Throughout this paper, S is a hemiring.

A non-empty subset A in S is called a *subhemiring* of S if A is closed under addition and multiplication. A non-empty subset A in S is called a *left* (resp. *right*) *ideal* of S if A is closed under addition and $SA \subseteq A$ (resp. $AS \subseteq A$). Further, A is called an *ideal* of S if it is both a left ideal and a right ideal of S.

An ideal I of S is called a k-ideal of S if $x \in S$, $a, b \in I$ and $x + a = b$ implies $x \in I$. An ideal I of S is called an h-ideal if $x, z \in S$, $a, b \in I$ and $x + a + z = b + z$ implies $x \in I$.

Definition 2.1 *([34])*. A pair $\mathfrak{S} = (F, A)$ is called a *soft set* over U, where $A \subseteq E$ and $F : A \rightarrow \mathcal{P}(U)$ is a set-valued mapping.

Definition 2.2 *([18])*. A soft set $\mathfrak{S} = (F, A)$ over U is called a *full soft set* if $\bigcup_{a \in A} F(a) = U$.

Definition 2.3 *([35])*. Let R be an equivalence relation on the universe U, (U, R) be a Pawlak approximation space. A subset $X \subseteq U$ is called *definable* if $R_* X = R^* X$; in the opposite case, i.e., if $R_* X - R^* X \neq \emptyset$, X is said to be a *rough set*, where two operations are defined as:

$$R_* X = \{x \in U : [x]_R \subseteq X\},$$

$$R^* X = \{x \in U : [x]_R \cap X \neq \emptyset\}.$$

Theorem 2.4 ([36]). *Suppose that (U, R) is a Pawlak approximation space and $A, B \subseteq U$. Then*

(1) $R_*(A) \subseteq A \subseteq R^*(A)$,
(2) $R_*(\emptyset) = \emptyset = R^*(\emptyset)$,
(3) $R_*(U) = U = R^*(U)$,
(4) $R_*(R_*(A)) = R_*(A)$,
(5) $R^*(R^*(A)) = R^*(A)$,
(6) $R^*(R_*(A)) = R_*(A)$,
(7) $R_*(R^*(A)) = R^*(A)$,
(8) $R_*(A) = (R^*(A^c))^c$,
(9) $R^*(A) = (R_*(A^c))^c$,
(10) $R_*(A \cap B) = R_*(A) \cap R_*(B)$,
(11) $R^*(A \cap B) \subseteq R^*(A) \cap R^*(B)$,
(12) $R_*(A \cup B) \supseteq R_*(A) \cap R_*(B)$,
(13) $R^*(A \cup B) = R^*(A) \cup R^*(B)$,
(14) $A \subseteq B \rightarrow R_*(A) \subseteq R_*(B)$, $R^*(A) \subseteq R^*(B)$.

Definition 2.5 *([17])*. Let $\mathfrak{S} = (F, A)$ be a soft set over U. Then the pair $P = (U, \mathfrak{S})$ is called a *soft approximation space*. Based on P, we define the following two operations:

$$\underline{apr}_P(X) = \{u \in U | \exists a \in A [u \in F(a) \subseteq X]\},$$

$$\overline{apr}_P(X) = \{u \in U | \exists a \in A [u \in F(a), F(a) \cap X \neq \emptyset]\},$$

assigning to every subset $X \subseteq U$ two sets $\underline{apr}_P(X)$ and $\overline{apr}_P(X)$ called the *lower* and *upper soft rough approximations* of X in P, respectively. If $\underline{apr}_P(X) = \overline{apr}_P(X)$, X is said to be soft definable; otherwise X is called a *soft rough set*. In what follows, we call it *Feng-soft rough set*.

Remark 2.6. In order to resolve theoretical and practical aspects, we usually require the soft set to be full in the above definition. If not, it is often limited the research value by means of Feng-soft rough sets, which can be found in the following example.

Example 2.7. Assume that $\mathfrak{S} = (F, A)$ is a soft set over U which is given by Table 1.

and $P = (U, \mathfrak{S})$ is a soft approximation space, we can see that the $\mathfrak{S}$ is not full.

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Please cite this article in press as: J. Zhan, et al., A novel soft rough set: Soft rough hemirings and corresponding multicriteria group decision making, Appl. Soft Comput. J. (2016), http://dx.doi.org/10.1016/j.asoc.2016.09.012

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