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ASOC 3673 1–16
ARTICLE IN PRESS
Applied Soft Computing xxx (2016) xxx–xxx
Contents lists available at ScienceDirect
Applied Soft Computing
journal homepage: www.elsevier.com/locate/asoc

Algorithms for interval-valued fuzzy soft sets in stochastic multi-criteria decision making based on regret theory and prospect theory with combined weight

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A R T I C L E I N F O

Article history:
Received 10 October 2015
Received in revised form 8 April 2016
Accepted 24 June 2016
Available online xxx

Keywords:
Interval-valued fuzzy soft set
Distance measure
Combined weight
Regret theory
Prospect theory

A B S T R A C T

This paper presents two novel interval-valued fuzzy soft set approaches. First, we initiate a new axiomatic definition of interval-valued fuzzy distance measure, which is expressed by interval-valued fuzzy number (IVFN) that will reduce the information loss and remain more original information. Then, the objective weights of various parameters are determined via normal distribution. Combining objective weights with subjective weights, we present the combined weights, which can reflect both the subjective considerations of the decision maker and the objective information. Later, we propose two algorithms to solve stochastic multi-criteria decision making problem, which take regret aversion and prospect preference of decision makers into consideration in the decision process. Finally, the effectiveness and feasibility of two approaches are demonstrated by two numerical examples.

1. Introduction

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Many complex issues in environmental science, engineering, medical science, and economics involve vagueness and fuzziness. While a wide variety of existing theories such as probability theory, fuzzy set theory [1], rough set theory [2], and interval mathematics [3] have been developed to model incertitude. However, each of these theories has its inherent difficulties as pointed out in [4]. The soft set theory, initiated by Molodtsov [4], is free from the inadequacy of the parameterized tools of those theories [1–3].

Current works on soft set are developing rapidly. The study of hybrid models combing soft sets with other mathematical structures is an important research topic. Maji et al. [5] firstly explored fuzzy soft sets, a more generalized notion combining fuzzy sets and soft sets. Yang et al. [6] developed the concept of interval-valued fuzzy soft sets. Peng et al. [7] presented Pythagorean fuzzy soft sets, and discussed their operations. Yang et al. [8] proposed the multi-fuzzy soft sets and successfully applied them to decision making, meanwhile they extended the multi-fuzzy soft sets to that of bipolar multi-fuzzy soft sets [9] which can describe the parameter more accurately and precisely. Wang et al. [10] initiated the hesitant fuzzy soft sets by integrating hesitant fuzzy set [11] with soft set model, and presented an algorithm to solve decision making problems. Xiao et al. [12] presented trapezoidal fuzzy soft set. Feng et al. [13] established a colorful connection between rough sets and soft sets. Jun [14,15] studied the application of soft sets in BCK/BCI-algebras and initiated soft BCK/BCI-algebras. Aktaş and Çağman [16] gave a definition of soft groups, and derived their basic properties using Molodtsov's definition of the soft sets. Yamak et al. [17] proposed soft hyperstructure and studied some properties of soft subhypergroupoids. Çağman et al. [18] defined the soft topology on a soft set, and presented its related properties. Moreover, there have been some applications of interval-valued fuzzy soft sets such as parameter reduction [19], clustering analysis [20], decision making [21,22].

The above decision models and methods are mainly based on the assumptions of full rationality. However, in practical decision making, people do not behave in a completely rational manner, and the obvious deviations between the actual decision making behavior and predictable values of the expected utility theory will be appeared. In 1947, Simon [23] proposed the principle of "bounded rationality", and he thought that human beings only had a bounded rationality for the decision making. Kahneman and Tversky [24] collected many individual behavior researches by investigations and experiments on the basis of Simon's "bounded rationality", and initiated the prospect

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http://dx.doi.org/10.1016/j.asoc.2016.06.036
1568-4946/

Please cite this article in press as: X. Peng, Y. Yang, Algorithms for interval-valued fuzzy soft sets in stochastic multi-criteria decision making based on regret theory and prospect theory with combined weight, Appl. Soft Comput. J. (2016), http://dx.doi.org/10.1016/j.asoc.2016.06.036

theory. To be clear, the decision maker always overvalues the small probability event and ignores the normal event, and he is more sensitive to losses than gains. Therefore, the decision making based on prospect theory is more in accordance with real decision making behavior and it has become a research topic recently [25–28].

Regret theory, initiated by Bell [29] and Loomes and Sugden [30], is one of the most popular nonexpected utility models. The core idea is that the decision makers not only pay attention to the results obtained by the choice of the alternative, but also pay attention to the outcome of other alternatives, and avoid choosing the alternative that will make them regret [29,30]. Moreover, there have been some applications of regret theory such as investment choices [31,32], route choice [33], auctions [34], decision making [35].

Stochastic multi-criteria decision making (SMCDM) is one of the most important styles of multi-criteria decision making (MCDM), which is characterized by the existence of multiple natural states in the decision making and cannot determine what states will appear, but can be estimated in advance of each the probability of the occurrence of the natural state, and the parameter values of alternative are different in different natural states. So stochastic multi-criteria decision making has a wide range of practical background. To the best of our knowledge, however, the study of the interval-valued fuzzy soft SMCDM problems based on regret theory and prospect theory has not been reported in the existing academic literature.

In order to compute the distance of two interval-valued fuzzy sets, we propose a new axiomatic definition and distance measure, which takes in the form of interval-valued fuzzy set. Comparing with the existing literature [36–39], our distance measure can remain more original decision information.

Considering that different sets of criteria weights will influence the ranking results of alternatives, we develop a novel method to determine the criteria weights by combining the subjective factors with the objective ones. This model is different from the existing methods, which can be divided into two categories: one is the subjective weighting methods and the other is the objective weighting methods, which can be computed by normal distribution [40]. The subjective weighting methods pay much attention to the preference information of the decision maker [22,41], while they neglect the objective information. The objective weighting methods do not take into account the preference of the decision maker, in particular, these methods fail to take into account the risk attitude of the decision maker [21,42,43]. The characteristic of our weighting model can reflect both the subjective considerations of the decision maker and the objective information. Consequently, combining subjective weights with objective weights, we provide a combined model to determine criteria weights.

To facilitate our discussion, we first review some background on soft sets, interval-valued fuzzy sets, interval-valued fuzzy soft sets, regret theory and prospect theory in Section 2. In Section 3, we introduce a new interval-valued fuzzy distance, which takes in the form of interval-valued fuzzy set. In Section 4, two approaches to interval-valued fuzzy soft sets in SMCDM based on regret theory and prospect theory are designed. In Section 5, two examples are presented to verify the proposed methods. Finally, conclusions are pointed out in Section 6.

2. Preliminaries

In this section, we will briefly recall the basic concepts of soft sets, fuzzy soft sets, interval-valued fuzzy soft sets, regret theory and prospect theory. See especially [4–6,24,29,30] for further details and background.

2.1. Basic concepts of soft sets, fuzzy soft sets, interval-valued fuzzy sets and interval-valued fuzzy soft sets

Definition 2.1. ([4]) Let $P(U)$ be the set of all subsets of U. A pair $(\widetilde{F}, A)$ is called a soft set over U, where $\widetilde{F}$ is a mapping given by $\widetilde{F} : A \to P(U)$.

Definition 2.2. ([5]) Let $\widehat{P}(U)$ be the set of all fuzzy subsets of U. A pair $(\widehat{F}, A)$ is called a fuzzy soft set over U, where F is a mapping given by $\widehat{F} : A \to \widehat{P}(U)$.

Definition 2.3. ([44]) An interval-valued fuzzy set (IVFS) I in U is given by

$$I = \{< x, [I^-(x), I^+(x)] > | \; x \in U\}, \tag{1}$$

where $0 \le I^-(x) \le I^+(x) \le 1$. For simplicity, we call $i = [I^-(x), I^+(x)]$ an interval-valued fuzzy number (IVFN) denoted by $i = [i^-, i^+]$.

Definition 2.4. ([45]) Let $x = [x^-, x^+]$ and $y = [y^-, y^+]$ be two IVFNs, $\lambda \in [0, 1]$, then their operational laws are defined as follows:

(1) $x + y = [x^-, x^+] + [y^-, y^+] = [x^- + y^-, x^+ + y^+]$;
(2) $\lambda x = [\lambda x^-, \lambda x^+]$;
(3) $x^\lambda = [(x^-)^\lambda, (x^+)^\lambda]$;
(4) $x \cdot y = [x^-, x^+] \cdot [y^-, y^+] = [x^- \cdot y^-, x^+ \cdot y^+]$;
(5) $x = y$ iff $x^- = y^-$ and $x^+ = y^+$;
(6) $x^c = [1 - x^+, 1 - x^-]$;
(7) $-x = [{}^* - x^+, -x^-]$.

From (1), we can see that it may be not satisfied the definition of IVFN when $x = [0.3, 0.4]$, $y = [0.7, 0.8]$, but it has significance of computation. Meanwhile, for (7), we know that it is out of positive number range, but it also has significance of computation. We can see IVFN as a special interval number which can be satisfied the above condition.

Definition 2.5. ([6]) A pair (F, A) is called an interval-valued fuzzy soft set over U, where F is a mapping given by $F : A \to \widetilde{P}(U)$. $\widetilde{P}(U)$ denotes the set of all interval-valued fuzzy subsets of U.

Let $x \in U$ and $e \in A$. $F(e)$ is an interval-valued fuzzy subset of U, and it is called an interval-valued fuzzy value set of parameter e. Let $F(e)(x)$ denote the membership value that object x holds parameter e, then $F(e)$ can be written as an interval-valued fuzzy set that $F(e) = \{x/F(e)(x) \,|\, x \in U\} = \{x/[F^-(e)(x), F^+(e)(x)] \,|\, x \in U\}$.

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