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0–1 Linear programming methods for optimal normal and pseudo parameter reductions of soft sets

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ABSTRACT

This paper aims to find better algorithms for solving parameter reduction problems of soft sets and gives their potential applications. Firstly, we define the matrix of dominant support parameters and use it to explain the essential reasons for the different choice values of arbitrary pair of objects. Then we propose techniques for translating the normal and pseudo parameter reduction problems of soft sets into several equivalent 0–1 linear programming models, thus reduction problems of soft sets can be solved by any computational software for integer programming. Compared with the algorithm proposed by Ma et al., experimental results show that our method for normal parameter reduction is more efficient particularly when the number of parameters is big. At last we build a software system to show the potential applications of our methods in decision support.

1. Introduction

1.1. Soft set theory

A soft set is a special 0–1 valued information system, which was initiated as a new mathematical tool for dealing with uncertainties and vagueness by Molodtsov [1] in 1999. The theory of soft sets has potential applications in various fields like game theory, operations research, decision making and so on. Many contributions have been made by researchers to soft set theory. Some of these contributions are connected with algebraic structures [2–15]. Some are combined and compared with other mathematical theories designed for modeling various types of vague concepts [16–28]. Particulary, soft sets were combined with rough sets [29–33] and information systems [34]. These extended soft set models mainly focused on the study of operation concepts [35], decision making criterions [36–38] and their applications [39–47]. Soft sets with incomplete information have been studied in [48–50].

1.2. Related work in parameter reduction research of soft set and motivations

Efforts have been made towards issues concerning parameter reduction of soft sets. Chen et al. [51] pointed out that the conclusion of soft set reduction offered in [42] was incorrect, and then presented a new notion of parameterization reduction in soft sets. This notion was compared with the concepts of attribute reduction in rough set theory. Ali [52] proposed to delete only one parameter at each time in order to avoid the heavy searching work. Gong et al [53] developed parameters reduction concepts in bijective soft set decision system under fuzzy environments. The concept of normal parameter reduction was introduced in [54]. Han [55] proposed a method for compiling all normal parameter reductions of a soft set in a propositional logic formula. An algorithm for optimal normal parameter reduction was also developed in [54]. But the algorithm cost a great amount of computation. Ma et al. [56] pointed out an important property of normal parameter reduction of soft sets. Then this property was used for reducing the workload for finding candidate parameter sets. This method

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Table 1
Tabular representation of a soft set $S=(F, A)$.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	σ_S
u_1	0	1	1	1	0	0	1	1	5
u_2	0	0	1	1	1	1	1	1	6
u_3	0	0	0	1	1	1	0	0	3
u_4	0	1	0	1	0	0	1	0	3
u_5	0	1	0	1	0	0	1	0	3
u_6	1	0	1	1	1	0	1	0	5

is based on enumeration such that it works slowly when the number of parameters is big. What is more, it is not suit for pseudo parameter reduction of soft sets. This is because the property used is not true anymore for pseudo parameter reduction.

The above existing methods for computing parameter reductions are enumeration algorithms. They are based on the initial definitions of parameter reductions: firstly an arbitrary subset of parameters is chose, then it is checked whether the conditions of parameter reductions are satisfied. The parameter reduction problem of soft sets can be regarded as a constraint satisfaction problem. The existing algorithms have not paid much attention to a suitable representation of this satisfaction problem.

In fact, the parameter reduction problems of soft sets can be translated as 0–1 liner programming problems. Han [57] proposed one kind 0–1 liner programming model for solving normal parameter reduction of soft set, where parameters are regarded also as 0–1 valued constraint variables. It was shown that the conditions for a parameter reduction can be represented by some linear constraints among local parameters.

In this paper we will go on investigating the linear methods for solving parameter reduction problems of soft sets. It is different from [57] in the following aspects: (1) We will study the properties of dominating sets of parameters. (2) We will propose another four kinds of 0–1 linear programming models. The method in [57] is a special situation of one of these four models. (3) We will also develop linear programming models for pseudo parameter reduction problems of soft sets. (4) Experimental results will be given for showing the efficiencies of our models.

The remainder of this paper is organized as follows. Section 2 introduces parameter reduction concepts in soft set theory. Section 3 makes characterizations for normal and pseudo parameter reductions of soft sets. Section 4 develops techniques for translating pseudo and normal parameter reductions of soft sets equivalently into 0–1 linear programming problems. Section 5 presents several comparison results between Ma et al.'s method and our 0–1 linear programming method. Section 6 shows the potential applications of our methods for the proposed questions. Finally, we come to a conclusion of this article and outlook for potential future work.

2. Preliminaries

In this paper, suppose $U=\{u_1, u_2, \ldots, u_n\}$ is a finite set of objects, E is a set of parameters. For example, the attributes in information systems can be taken as parameters. $\wp(U)$ means the powerset of U, $|A|$ means the cardinality of set A. By [1] and [38] we have basic concepts about soft sets shown in Definitions 2.1 and 2.2.

Definition 2.1. [Soft set] A soft set on U is a pair $S=(F, A)$, where

(i) A is a subset of E;
(ii) $F: A \to \wp(U)$, $\forall e \in A$, $F(e)$ means the subset of U corresponding with parameter e. We also use $F(u, e)=1$ ($F(u, e)=0$) to mean than u is (not) an element of $F(e)$.

Definition 2.2. [Support set of parameters for objects] Let $S=(F, A)$ be a soft set over U. $\forall u \in U$, define the support set of parameters for u as the set $\{e \in A | F(u, e)=1\}$, denoted by $supp(u)$.

Definition 2.3. [Choice value function] Let $S=(F, A)$ be a soft set over U. The function $\sigma_S : U \to \mathbb{N}$ defined by $\sigma_S(u)=|supp(u)|=\sum_{e\in A}F(u, e)$ is called the choice value function of S.

We write σ_S as σ for short if the underlying soft set S is explicit.

Example 2.1. Table 1 represents a soft set $S=(F, E)$ over objects domain $U=\{u_1, u_2, \ldots, u_6\}$ and parameters domain $E=\{e_1, e_2, \ldots, e_8\}$, where $F(e_1)=\{u_6\}$, $F(e_2)=\{u_1, u_4, u_5\}$, $F(e_3)=\{u_1, u_2, u_6\}$, $F(e_4)=U$, $F(e_5)=\{u_2, u_3, u_6\}$, $F(e_6)=\{u_2, u_3\}$, $F(e_7)=\{u_1, u_2, u_4, u_5, u_6\}$, $F(e_8)=\{u_1, u_2\}$. σ_S can be regarded as the choice value function of soft set S.

According to [54] and [56], we have the following concepts about parameter reduction of soft sets.

Definition 2.4. [Normal parameter reduction] For soft set $S=(F, A)$ over U, $B \subseteq A$, $B \neq \emptyset$, if the constraint $\sum_{e\in A-B}F(u_1, e)=\cdots=\sum_{e\in A-B}F(u_n, e)$ (denoted by C_{normal}) is satisfied, then B is called a normal parameter reduction of S.

Definition 2.5. [Pseudo parameter reduction] For soft set $S=(F, A)$ over U, $B \subseteq A$, $B \neq \emptyset$, assume $S'=(F, B)$, denote the choice value function of S' as σ'. If the conjunction of the following two constraints (denoted by C_{pseudo}) is satisfied:

(i) $\sigma(u)=\sigma(v)$ implies $\sigma'(u)=\sigma'(v)$;
(ii) $\sigma(u)>\sigma(v)$ implies $\sigma'(u)>\sigma'(v)$, then B is called a pseudo parameter reduction of S.

For soft set $S=(F, A)$ over U, denote the set of all pseudo parameter reductions of S by $PPR(S)$; denote the set of all normal parameter reductions of S by $NPR(S)$. It is obvious that $NPR(S) \subseteq PPR(S)$.