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Two population-based optimization algorithms for minimum weight connected dominating set problem

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ABSTRACT

Minimum weight connected dominating set (MWCDS) is a very important NP-Hard problem used in many applications such as backbone formation, data aggregation, routing and scheduling in wireless ad hoc and sensor networks. Population-based approaches are very useful to solve NP-Hard optimization problems. In this study, a hybrid genetic algorithm (HGA) and a population-based iterated greedy (PBIG) algorithm for MWCDS problem are proposed. To the best of our knowledge, the proposed algorithms are the first population-based algorithms to solve MWCDS problem on undirected graphs. HGA is a steady-state procedure which incorporates a greedy heuristic with a genetic search. PBIG algorithm refines the population by partially destroying and greedily reconstructing individual solutions. We compare the performance of the proposed algorithms with other greedy heuristics and brute force methods through extensive simulations. We show that our proposed algorithms perform very well in terms of MWCDS solution quality and CPU time.

1. Introduction

The dominating set (DS)[1] and its variants are popular graph theoretic structures which are used in many applications such as clustering, backbone formation and intrusion detection in wireless ad hoc and sensor networks (WASNs) [1–3], gateway placement in wireless mesh networks [4], deployment of wavelength division multiplexing in optical networks [5], information retrieval for multi-document summarization [6] and query selection for obtaining data from web databases [7].

For a given undirected graph (UG) $G(V, E)$where all edges are bidirectional, V is the set of vertices and E is the set of edges; the minimum dominating set (MDS) problem is to find a subset of vertices $D \subseteq V$ where each node in $V \setminus D$ is adjacent to at least one node in D. The nodes in D and $V \setminus D$ are called as dominators and dominatees, respectively. Finding the minimum set of dominators for a given undirected graph is an NP-Hard problem. An example application of MDS problem is clustering a WASN where dominators are cluster heads and dominatees are cluster members. If D is a DS and each node pair $(v_i,v_j) \in D$ has at least a path that consists of only nodes in D, then the D is defined as the connected dominating set (CDS). CDS is a very useful structure for backbone formation in WASNs [2] such as data collected from dominatees are relayed by the dominators through CDS to the sink node. Similar to the MDS problem, finding the minimum CDS (MCDS) is an NP-Hard problem. Energy efficient operation is of utmost importance in WASNs since generally nodes are battery-powered. It is a well-known fact that the communication is the dominant factor of the energy consumption [8]. Hence, the nodes in the CDS backbone may exhaust their batteries very earlier than others since they are responsible for carrying the data transmission. One of the solutions of this problem is selecting the nodes with high energies as dominators. To achieve this, a weighted connected dominating set (WCDS) backbone has been applied [9] in which the total weight of CDS is aimed to be minimized. Same as its unweighted version, finding the minimum WCDS (MWCDS) is in NP-Hard complexity class.

There are approximation algorithms [9,10] based on heuristics to solve the MWCDS problem on unit disk graphs (UDGs) which are used to model WASNs. Although UDG can be an appropriate model to use the inherent geometrical properties of the ideal wireless communication, the transmission range of a node may not be circular in some cases such as a network area that includes obstacles [11]. Hence, UG is a better model in this situation.

In this paper, we propose two population-based optimization algorithms for MWCDS problem on UG. To the best of our knowledge, these are the first population-based algorithms proposed for MWCDS on UGs. Our first approach for solving this problem is a hybrid genetic algorithm (HGA). This algorithm is a heuristic based steady-state genetic algorithm which gives favorable results for

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[1] The acronyms used throughout the text are explained in Table 1.

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similar combinatorial graph theoretical problems such as MDS [12] and minimum vertex cover problems [13]. Our second approach is a population-based iterated greedy (PBIG) algorithm which has been recently received significant attention for tackling various optimization problems [14–18]. PBIG is based on iterated greedy (IG) technique which belongs to a class of stochastic local search consisting of several methods such as iterative improvement, iterated local search and ant colony optimization (ACO) [19]. Rather than improving single solution, the proposed algorithms are population-based optimization algorithms which work on a set of solutions. To select dominators in both proposed approaches, we combine the greedy CDS heuristic of Guha and Khuller [20] and set cover based heuristic of Chvatal [21]. For the minimization operation in HGA and the destruction phase in PBIG, we do not remove cut vertices from WCDS with regarding the fact that a MWCDS always includes cut vertices in order to provide connectivity between dominators.

We evaluate the performance of our algorithms by comparing with other algorithms in terms of total weight of the produced WCDS and wall clock times. We use the benchmark instances given in [22] and generate our own dataset to test the proposed algorithms extensively. Regarding to the performed experiments, the proposed algorithms perform very well both in terms of MWCDS solution quality and time taken.

The rest of the paper is organized as follows: Section 2 provides preliminaries. Section 3 gives related work about WCDS. Section 4 explains greedy heuristics. The description of the proposed HGA and the complexity analysis are given in Section 5. Section 6 describes the steps of the proposed PBIG algorithm in detail and provides the complexity considerations. The comparisons of the results with different methods are discussed in Section 7. Finally, Section 8 draws the conclusions.

2. Preliminaries

A cut vertex (articulation point) is a vertex whose removal partitions the graph into disconnected components. In Fig. 1.a, an example UG with 8 nodes is given. Node D which is colored black is a cut vertex for this example where its removal divides the network into two disconnected components as: {A, B, C} and {E, F, G, H}.

Given a vertex set $V' \subseteq V$, a vertex induced subgraph by V' is $G' = (V', E')$ where E' is the set of all edges $\{(v_1, v_2): ((v_1, v_2) \in E) \bigwedge (v_1 \in V') \bigwedge (v_2 \in V')\}$. Fig. 1.b shows an example vertex induced subgraph G' from G given that $V' = \{B, D, E, F\}$. G' is a linear graph in which nodes D and E are cut vertices.

Hopcroft and Tarjan's algorithm detects cut vertices in an undirected graph [23]. This algorithm runs in linear time and uses depth-first search (DFS) algorithm. The parent of node u (*parent(u)*) in DFS tree is node v which has been just visited before node u. The depth value of the root node is set to 1. The depth of node u (*depth(u)*) is calculated as *depth(u)* = *depth(parent(u))* + 1. The low value (*low(u)*) is calculated as the min{*depth(u)*, the depth values of node u's neighbors other than its children and *parent(u)*, the low values of children of node u}. The *depth(u)* shows the distance of node u to root in DFS there, whereas the *low(u)* indicates the smallest depth of the node reachable from subtree rooted with u. The steps of the algorithm are as follows:

1. A DFS algorithm is first applied.
2. A non-root node u is a cut vertex if it has a child node v which satisfies $low(v) \geq depth(u)$.
3. The root node is a cut vertex if it has at least two children in its DFS tree.

The time complexity of this approach is same with DFS and equals to $O(V + E)$. Fig. 1.c shows an example operation of this algorithm on graph G. The root node is A which has an extra circle. A directed edge shows a parent→child relationship. The dashed edges are the edges existing on G but do not belong to the DFS tree. The depth and low values are shown with d and l, respectively. The DFS starts from node A and proceeds B, D and C, sequentially. The depth values of A, B, D and C are 1, 2, 3, and 4, respectively. Since node C does not have any unvisited neighbor, DFS backtracks to node D. Then, nodes E, F, G and H are visited where the depth values of these nodes are 4, 5, 6, and 7, respectively. The low value of node H is 3 because it is directly connected to node D and there is no parent→child relationship between them. The low values of G, F and E are 3 since their subtrees include node H. Node C's low value is 1 since the depth of node A is 1. The low values of nodes B and D are 1 because their subtrees include node C. Node A's low value is 1 because its depth is 1. Since the low value of node E is 3 and that is equal to node D's depth value, node D is a cut vertex in this graph. There is no another node satisfying this condition.

MWCDS problem can be formally defined as follows. For a given node weighted graph $G = (V, E, w)$ where w is a function $w: V \rightarrow R^+$, the MWCDS problem is to find a CDS with the minimum total weight $W_T(V) = \sum_{v_i \in D} w(v_i)$. Throughout this paper, $\Gamma(v)$ indicates the open neighborhood of node v such that $\Gamma(v) = \{u \in V : (u, v) \in E\}$. The closed neighborhood set of node v consists of the union of node v's open neighborhood and itself, and it can be defined as $\Gamma[v] = \Gamma(v) \cup \{v\}$. We use colors to classify nodes where BLACK is used for a dominator node v (color(v) = BLACK), GRAY and WHITE are used for a dominatee node. A GRAY node differs from a WHITE node that it has at least one BLACK neighbor. We use $\Gamma(v)_c$ to show the node v's open neighborhood with color c and it is formally defined as $\Gamma(v)_c = \{u \in \Gamma(v) : color(u) = c\}$. In a similar manner, $\Gamma[v]_c = \{u \in \Gamma[v] : color(u) = c\}$ is used to show the node v's closed neighborhood with color c.

3. Related work

Various algorithms are proposed to find minimal and efficient DSs [12,21,24,25]. Chavatal [21] proposes a greedy heuristic for the set-covering related problems in which a ratio is calculated for each set P_j as $|P_j|/C_j$ where $|P_j|$ is the number of covered points and C_j is the cost. The set having the minimum ratio is selected in each step until all points are covered. This heuristic can be applied to construct a weighted dominating set (WDS) problem and we apply this heuristic in our proposed algorithms. The approximation ratio of this approach for the minimum WDS (MWDS) is $O(\log W_T(S))$ where S is the optimum solution set. Jovanovic [24] proposes an ACO applied to the MWDS problem. The algorithm considers node weights and it is tested against the greedy algorithm under various graph sizes, edge densities and weight distribution functions. The results show that the algorithm outperforms the greedy approach on many test setups. Potluri and Singh [12] propose hybrid metaheuristic algorithms for constructing WDSs. The algorithms work on UGs and perform better than the greedy approach. An artificial bee colony algorithm is proposed for MWDS problem by Nitash and Singh [25] for UGs. They compare their algorithm with other metaheuristics in literature and claim that their algorithm is better than these metaheuristics. Bouamama and Blum [18] propose a randomized version of PBIG algorithm for the MWDS problem. Their experiments are conducted on Jovanovic's dataset in [24]. The first polynomial time approximation scheme for MWDS with smooth weights, which achieves a $(1 + \varepsilon)$-approximation for any $\varepsilon > 0$ is given in [26]. Lin [27] proposes a hybrid self-adaptive evolutionary algorithm for MWDS problem. The aim of the algorithms mentioned so far is to solve the MWDS problem, on the other side we study on the connected version of this problem MWCDS, thus they are out of our concern.