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A new time invariant fuzzy time series forecasting method based on particle swarm optimization

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ABSTRACT

In the analysis of time invariant fuzzy time series, fuzzy logic group relationships tables have been generally preferred for determination of fuzzy logic relationships. The reason of this is that it is not need to perform complex matrix operations when these tables are used. On the other hand, when fuzzy logic group relationships tables are exploited, membership values of fuzzy sets are ignored. Thus, in defiance of fuzzy set theory, fuzzy sets' elements with the highest membership value are only considered. This situation causes information loss and decrease in the explanation power of the model. To deal with these problems, a novel time invariant fuzzy time series forecasting approach is proposed in this study. In the proposed method, membership values in the fuzzy relationship matrix are computed by using particle swarm optimization technique. The method suggested in this study is the first method proposed in the literature in which particle swarm optimization algorithm is used to determine fuzzy relations. In addition, in order to increase forecasting accuracy and make the proposed approach more systematic, the fuzzy *c*-means clustering method is used for fuzzification of time series in the proposed method. The proposed method is applied to well-known time series to show the forecasting performance of the method. These time series are also analyzed by using some other forecasting methods available in the literature. Then, the results obtained from the proposed method are compared to those produced by the other methods. It is observed that the proposed method gives the most accurate forecasts.

1. Introduction

In recent years, fuzzy time series method was firstly introduced by Song and Chissom [30]. The method has been widely used in the literature since it does not require any assumption. Fuzzy time series can be split into two subclasses which are time variant and time invariant. In time invariant fuzzy time series, it is assumed that auto correlations do not change due to the time. However, it is assumed that auto correlations change due to the time in time variant ones. Fuzzy time series methods generally consist of three main phases such as fuzzification, determination of fuzzy logic relationships, and defuzzification. In the literature, new methods have been proposed by making contributions to these phases.

There have been various methods suggested for fuzzification phase of fuzzy time series. Song and Chissom [31–33], Chen [6], Chen [7], Tsaur [35], Singh [30], Egrioglu et al. [12,13] used fuzzification methods based on partition of universe of discourse. The operation of partition of universe of discourse is to divide universe of discourse into equal or unequal sub-intervals according to a predefined interval length. The effect of the predefined interval length on the forecasting performance was examined by Huarng [18]. Huarng [18] also suggested two methods such as mean and distribution based approaches to determine the length of interval. Huarng and Yu [19] suggested using a dynamic interval length, which is increased due to a ratio value, instead of a static one. Yolcu et al. [36] improved the ratio based method proposed by Huarng and Yu [19] by using optimization to determine the ratio value. Later on, the idea of dynamic length of interval has been also used in other studies in the fuzzy time series literature. Davari et al. [11], Kuo et al. [22,23], Park et al. [28], Hsu et al. [16], Fu et al. [15], and Huang et al. [17] utilized particle swarm optimization technique to determine the dynamic length of interval. Lee et al. [24,25] also utilized genetic algorithm for the same purpose.

Another way for fuzzification is to employ a clustering method. Cheng et al. [9] and Li et al. [26] used the fuzzy *c*-means clustering method in the fuzzification phase. While hierarchical clustering algorithms which are non-fuzzy are employed in Chen and Tanuwijaya [8], and Bang and Lee [4], Gustafson–Kessel fuzzy clustering method is used utilized in Egrioglu et al. [14] for fuzzification.

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In the phase of defuzzification, feed forward neural networks are employed by Song and Chissom [33]. Chen [6], Huarng [18], and Huarng and Yu [19] used centroid method for defuzzification. Cheng et al. [10] and Aladag et al. [2] preferred to use adaptive expectation method in the defuzzification process.

In time invariant fuzzy time series, fuzzy logic relationship can be expressed by only one fuzzy logic relation matrix. Sullivan and Woodall [34] used transition matrices based on Markov chain instead of using fuzzy logic relation matrix. Chen [6] proposed an approach that makes the phase of determination of fuzzy relations easier. In the method proposed by Chen [6], instead of using fuzzy logic relation matrix, fuzzy logic group relationships tables are used to establish fuzzy logic relationships. Also, fuzzy logic group relationships tables have been used in many studies such as Huarng [18], Yu [38], Huarng and Yu [19], Cheng et al. [10], and Egrioglu et al. [14]. Huarng and Yu [20] suggested a method in which feed forward neural networks are used to determine fuzzy relations. A high order fuzzy time series forecasting method in which feed forward neural networks are employed to establish fuzzy relations was proposed by Aladag et al. [1]. In Aladag et al. [3], Elman recurrent neural networks were also used to determine fuzzy relations. Yu and Huarng [39], and Yolcu et al. [37] proposed a different approach in which membership function values are employ to use feed forward neural networks in the phase of determination of fuzzy relations.

In the analysis of time invariant fuzzy time series, the phase of determination of fuzzy logic relationships has important impact on forecasting performance. By establishing fuzzy logic relations, Song and Chissom [31] obtained a relationship matrix consists of these fuzzy logic relations. Obtaining a relationship matrix requires complex operations. Thus, Chen [6] proposed to use fuzzy logic group relationships tables in the phase of determination of fuzzy logic relationships differently from the method proposed by Song and Chissom [31]. Instead of complex matrix operations, fuzzy logic group relationships tables have been generally preferred in the literature to establish fuzzy relations. However, when fuzzy logic group relationships tables are used, membership values of fuzzy sets are ignored and in defiance of fuzzy set theory, fuzzy sets' elements whose membership value is 1 are only taken into account. This situation causes information loss and decrease in the explanation power of the model. To overcome these problems, particle swarm optimization method is used to calculate membership values in the fuzzy relationship matrix in the time invariant fuzzy time series approach proposed in this study. The method suggested in this study is the first method proposed in the literature in which particle swarm optimization algorithm is used to determine fuzzy relations. In addition to this, the fuzzy c-means clustering method is employed for fuzzification in the proposed method. The decision of the number of intervals, arbitrary determination of interval length, and arbitrary choice of degree of membership are some problems arise in the fuzzification phase. In order to overcome these problems, the fuzzy c-means clustering method is employed in the proposed method. Therefore, the proposed time invariant fuzzy time series forecasting approach works in a systematic way and more accurate forecasts can be obtained when the proposed model is employed. To show the performance of the proposed method, it is applied to three data sets which are well known in the fuzzy time series literature. The enrollment data of University of Alabama, Index 100 in stocks and bonds exchange market of İstanbul, and TAIFEX (Taiwan Futures Exchange) data are employed in the implementation. The results obtained from the proposed method are compared to those produced by other fuzzy time series forecasting approaches available in the literature.

Basic definitions of fuzzy time series are given in the next section. The fuzzy c-means clustering and particle swarm optimization methods are briefly presented in Sections 3 and 4, respectively. Section 5 introduces the proposed method. The implementation is given in Section 6. Finally, the last section concludes the paper.

2. Preliminaries on fuzzy time series

The fuzzy time series was firstly defined by Song and Chissom [31]. The basic definitions of fuzzy time series, and time variant and time invariant fuzzy time series definitions are given below.

Definition 1. Let $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots)$, a subset of real numbers, be the universe of discourse on which fuzzy sets $f_j(t)$ are defined. If $F(t)$ is a collection of $f_1(t), f_2(t), \ldots$ then $F(t)$ is called a fuzzy time series defined on $Y(t)$.

Definition 2. Suppose $F(t)$ is caused by $F(t-1)$ only, i.e., $F(t-1) \to F(t)$. Then this relation can be expressed as $F(t) = F(t-1) \circ R(t, t-1)$ where $R(t, t-1)$ is the fuzzy relationship between $F(t-1)$ and $F(t)$, and $F(t) = F(t-1) \circ R(t, t-1)$ is called the first order model of $F(t)$. "$\circ$" represents max–min composition of fuzzy sets.

Definition 3. Suppose $R(t, t-1)$ is a first order model of $F(t)$. If for any t, $R(t, t-1)$ is independent of t, i.e., for any t, $R(t, t-1) = R(t-1, t-2)$, then $F(t)$ is called a time invariant fuzzy time series otherwise it is called a time variant fuzzy time series.

Song and Chissom [30] firstly introduced an algorithm based on the first order model for forecasting time invariant $F(t)$. In [30], the fuzzy relationship matrix $R(t, t-1) = R$ is obtained by many matrix operations. The fuzzy forecasts are obtained based on max–min composition as below:

$$F(t) = F(t-1) \circ R \tag{1}$$

The dimension of R matrix depends on the number of fuzzy sets. The number of fuzzy sets equals to the number of intervals that compose of universe of discourse. The more fuzzy sets are used, the more matrix operations are needed for obtain R matrix. When the number of fuzzy set is high, using the method proposed by Song and Chissom [32] considerably increases the computational cost.

3. The fuzzy c-means clustering method

In the method proposed by Song and Chissom [31], partition of universe of discourse method is used in the fuzzification phase. However, there are several problems related to the decomposition of universe of discourse. These problems are that determination of the number of intervals, arbitrarily choice of interval length and membership degrees. In order to deal with these problems, Cheng et al. [9] and Li et al. [26] used fuzzy c-means clustering method for fuzzification. The fuzzy c-means clustering method was firstly introduced by Bezdek [5]. This clustering method is the most widely used clustering algorithm. In this method, fuzzy clustering is conducted by minimizing the least squared errors within groups. Let u_{ij} be the membership values, v_i be the center of cluster, n be the number of variables, and c be the number of clusters. Then the objective function, which is tried to be minimized in fuzzy clustering, is

$$J_\beta(X, V, U) = \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^{\beta} d^2(x_j, v_i) \tag{2}$$

where β is a constant $(\beta > 1)$ and called the fuzzy index. $d(x_j, v_i)$ is a similarity measure between an observation and the center of