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Uncertain Shapley value of coalitional game with application to supply chain alliance[☆]

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ABSTRACT

Uncertain coalitional game deals with situations in which the transferable payoffs are uncertain variables. The uncertain core has been proposed as the solution of uncertain coalitional game. This paper goes further by presenting two definitions of uncertain Shapley value: expected Shapley value and α-optimistic Shapley value. Meanwhile, some characterizations of the uncertain Shapley value are investigated. Finally, as an application, uncertain Shapley value is used to solve a profit allocation problem of supply chain alliance.

1. Introduction

Game theory has been used extensively in many fields since von Neumann and Morgenstern [28] established modern game theory in 1944. Distinguished from strategic game and extensive game, coalitional game focuses on the behavior of the group of players as a coalition, rather than between individual players. In literature, many researchers introduced different solution concepts of coalitional game from different perspectives (e.g., core by Aumann and Maschler [2], Shapley value by Shapley and Shubik [38], and nucleolus by Schmeidler [36]), among which the core and Shapley value are mostly used.

Consider a supply chain alliance that is composed by several independent entities to achieve common goals in a coexistence market environment. Such the alliances bring firms the advantages of enhancing cooperative behavior and resolving competitive conflicts, obtaining greater learning benefits, developing innovative products, dealing with turbulence and market uncertainty risk. Each cooperative enterprise in their respective areas of strength (such as design, manufacturing and retail, etc.) devotes to their core competencies to achieve complementary advantages, risk-sharing and benefit sharing for the supply chain alliance. Then, fair and reasonable mechanism of profit allocation is critical for the supply chain alliance. Shapley value is one way to distribute the total gains to the players. Suppose that each player demands his contribution as a fair compensation once a coalition is formed. Then it is "fair" for the player to take the average of his contributions (Shapley value) over the possible different permutations in which the coalition can be formed. Thus, Shapley value is the only "fair" distribution in the sense of contribution.

In real game situations, as Harsanyi [18] pointed out, "the player may lack full information about the other players' payoffs (or even their own), etc." Moreover, it is difficult to clarify the precise relationship between payoffs and different strategic profiles. For example, there are two petroleum companies in the market. If they don't cooperate, then their payoffs will be decreasing. In the case of collective rationality, they would form a coalition to create a dominant market-share for making more profit. But in such situation, the payoffs of the coalition and themselves are still unpredictable and indeterminable. This indeterminacy affects the prospective assigned payoffs when they form the coalition. Particularly, in a supply chain alliance, the players are in different areas like design, manufacturing and retail. Then, exact evaluation of the payoffs for different coalitions are almost impossible. So are the contribution of each player and Shapley value of the coalitional game. In order to estimate the payoffs of the coalition and themselves, we should rely on the subjective-intuitive opinions of experts with rich managerial experiences, because there is no past statistical evidence and the payoffs of different coalitions are very difficult to evaluate. For example, the payoffs of the coalition may be expressed by human

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language like "about ten million dollars". Then, how to deal with such subjective uncertainty is a basic problem for further discussion of coalitional game with indeterminacy.

In order to model subjective uncertainty, two mathematical systems are used, among others: fuzzy set theory that is based on fuzzy information, while another is uncertainty theory based on the human's belief degree. For scientists of fuzzy set theory, fuzzy set theory offers an appropriate and powerful alternative to deal with the incomplete information; and applications of fuzzy set can be found in references [20–22,40,41]. Usually, the quantities like "about ten million dollars" are assumed to be fuzzy variables. In literature, fuzzy coalitional game may be dated to Aubin [1] and Butnaria [3]. By assuming that the payoffs are fuzzy variables, Mareš [30], Nishizaki and Sakawa [34], Shen and Gao [39] and Gao et al. [11] discussed various solution concepts of the fuzzy-payoff coalitional game. While Mareš [31] introduced the fuzzy coalition structures based on the memberships of coalitions in a coalitional game. In addition, the fuzzy-payoff strategic game was discussed by many researchers (e.g., Campos and Gonzalez [4], Maeda [29], Nishizaki and Sakawa [33], Gao and his co-workers [8–10,12,23]). The fuzzy-payoff extensive game was also discussed by Gao and Yu [13]. For more detailed exposition of the games with fuzzy information, the reader may consult the books by Mareš [32], Nishizaki and Sakawa [35], and Gao and Yang [16].

For scientists of uncertainty theory, uncertainty theory offers another appropriate and powerful alternative to deal with the human's belief degree. And, the quantities like "about ten million dollars" are assumed to be uncertain variables. Uncertainty theory was proposed by Liu [24] in 2007 and refined by Liu [26] in 2010 to handle the belief degrees. Uncertain theory is based on normality, duality, subadditivity and product axioms, which distinguishs it from fuzzy set theory. Nowadays, uncertainty theory has become a new branch of mathematics including uncertain set [27,44,45], uncertain process [15,25,50,53,59], uncertain differential equation [5,51,54,55,58], etc. Uncertain theory was also applied to many areas like management [19,57], control [46,47,60], and finance [6,7,17,52,56], etc. By assuming that the payoffs are uncertain variables, Yang and Gao [43] presented two definitions of uncertain core as the solutions of uncertain coalitional game. Meanwhile, uncertain strategic game was also discussed by Gao [14] and Yang and Gao [48], uncertain differential game was also investigated by Yang and Gao [42,49].

In this work, we go further to study the Shapley value of the coalitional game with uncertain payoffs. Section 2 calls on the concepts of Shapley value and briefly reviews some basic results of uncertainty theory including two uncertain ranking approaches to define the behavior of players under uncertain situation. In Section 3, the expected Shapley value and the α-optimistic Shapley value are proposed as the solutions of the coalitional game with uncertain payoffs. Meanwhile, the uniqueness of the two uncertain Shapley values is proved. Finally, a profit allocation problem of supply chain alliance is provided to illustrate the usefulness of the theory developed in this paper.

2. Preliminaries

In this section, we first recall the notion of coalitional game with transferable payoffs, and then give some basic results in uncertainty theory.

2.1. The Shapley value

In the early 1953s, Shapley [37] introduced Shaley value that describes the payoff of players in a coalitional game. Because Shapley value is simple to be processed with mathematical methods, it is used widely in both economy and political science. Now we give the definition of coalition game with transferable payoff.

Definition 2.1 *(Shapley [37]).* A coalitional game $\langle N, v\rangle$ with transferable payoff consists of:

1. a finite set N (the set of the players);
2. a function v that associates a real number $v(S)$ (the payoff of S) with every nonempty subset S (a coalition) of N.

For each coalition S with transferable payoff, the payoff $v(S)$ is the total payoff that is available for the division among the members of S. An assumption of the model is that

$$v(S\cup T)\geq v(S)+v(T)$$

for all the S and T with $S\cap T=\emptyset$. It means that the payoff of a coalition must be more than sum of the payoff that each player could receive if he or she does not join the coalition. We call this condition as superadditivity.

Definition 2.2 *(Shapley [37]).* The Shapley value φ is defined by the condition

$$\varphi_i(N,v)=\frac{1}{|N|!}\sum_{S\subseteq N}(s-1)!(|N|-s)!(v(S)-v(S\backslash\{i\}))$$

where s is the number of players in the coalition S, $\forall i\in N$.

The value means the payoff of player i in a coalitional game $\langle N, v\rangle$. Then the payoff in this game is presented as a vector $\varphi(N,v)=(\varphi_i(N,v))_{i\in N}$. Obviously,

$$\sum_{i\in N}\varphi_i(N,v)=v(N).$$

And we call $\Delta_i(S)=v(S)-v(S\backslash\{i\})$ is marginal contribution of player i to coalition S.

In the following, some definitions and three axioms are given such that we can turn to an axiomatic characterization of Shapley value.

Definition 2.3 *(Shapley [37]).* Player i is a dummy in $\langle N, v\rangle$, if

$$v(S)-v(S\backslash\{i\})=v(\{i\})$$

for each coalition S that includes i.

Definition 2.4 *(Shapley [37]).* Player i and j are interchangeable in $\langle N, v\rangle$, if $v(S\backslash\{i\})=v(S\backslash\{j\})$, $\forall S\subseteq N$.

The three axioms are as follows,

1. SYM (Symmetry): If i and j are interchangeable in $\langle N, v\rangle$, then
$$\varphi_i(N,v)=\varphi_j(N,v);$$
2. DUM (Dummy player): If i is a dummy in $\langle N, v\rangle$, then
$$\varphi_i(N,v)=v(\{i\});$$
3. ADD (Additivity): For any two games v and w, there is
$$\varphi_i(v+w)=\varphi_i(v)+\varphi_i(w)$$
for all $i\in N$, where $(v+w)(S)=v(S)+w(S)$, $\forall S\subseteq N$.

Lemma 2.1 (Shapley [37]). *The Shapley value is the only value that satisfies SYM, DUM and ADD.*