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Optimal control for stochastic linear quadratic singular Takagi–Sugeno fuzzy delay system using genetic programming

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ABSTRACT

In this paper, optimal control for stochastic linear singular Takagi–Sugeno (T–S) fuzzy delay system with quadratic performance is obtained using genetic programming (GP). To obtain the optimal control, the solution of matrix Riccati differential equation (MRDE) is computed by solving differential algebraic equation (DAE) using a novel and nontraditional GP approach. The GP solution is equivalent or very close to the exact solution of the problem. Accuracy of the GP solution to the problem is qualitatively better. The solution of this novel method is compared with the traditional Runge Kutta (RK) method. An illustrative numerical example is presented for the proposed method.

1. Introduction

Time delay exists inevitably in active control systems which mainly results from the following:

(a) The time taken in the online data acquisition from sensors at different locations of the system.
(b) The time taken in the filtering and processing of the sensory data for the required control force calculation and the transmission of the control force to the actuator.
(c) The time taken by the actuator to produce the required control force.

Time delay systems occur in various fields such as aeronautical, astronautical, mechanical, chemical and electrical engineering. Many methods have been proposed to deal with the time delay control system [10].

A fuzzy system consists of linguistic IF–THEN rules that have fuzzy antecedent and consequent parts. It is a static nonlinear mapping from the input space to the output space. The inputs and outputs are crisp real numbers and not fuzzy sets. The fuzzification block converts the crisp inputs to fuzzy sets and then the inference mechanism uses the fuzzy rules to produce fuzzy conclusions or fuzzy aggregations and finally the defuzzification block converts these fuzzy conclusions into crisp outputs. The fuzzy system with singleton fuzzifier, product inference engine, center average defuzzifier and Gaussian membership functions is called as standard fuzzy system [30].

Two main advantages of fuzzy systems for the control and modelling applications are (i) uncertain or approximate reasoning, especially difficult to express a mathematical model (ii) decision making problems with the estimated values under incomplete or uncertain information [35,36].

Genetic programming is an evolutionary algorithm that attempts to evolve solution to the given problem by using concepts taken from naturally occurring evolving process. The technique is based on the evolution of a large number of candidate solutions through genetic operations such as reproduction, crossover and mutation. It is based upon the Genetic algorithm (GA) [14], which

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exploits the process of natural selection based on a fitness measure to breed a population of trial solution that improves over time. While GA usually operates on (coded) strings of numbers but GP uses the principles and ideas from biological evolution to guide the computer to acquire desired solution. The search space is too large to attempt a brute force search, the method must be utilized to reduce the number of examined solutions. In this search, initially the population looks a bit like a cloud of randomly selected points, but that generation after generation it moves in the search space following a well defined trajectory. The generation is achieved with the help of grammatical evolution, because grammatical evolution can produce programmes in an arbitrary language, the genetic operations are faster and also because it is more convenient to symbolically differentiate mathematical expression. The code production is performed using a mapping process governed by grammar expressed in Backus Naur Form (BNF) [23]. In analogy to nature, the potential solution is an individual in some collection or population of potential solutions. The individuals who are stronger, meaning higher ranked according to fitness function, will be used to determine the next collection of potential solution. A new generation will be arisen by employing analogs of reproduction and mutation.

This means that GP has advantages over other algorithms as it can perform optimization at a structural level. This enabled Koza [17] to demonstrate the application of GP algorithm to a number of problem domains, including regression, control and classification. Research in this area has grown rapidly and encompassed a wide range of problems. GP techniques have been successfully applied in various engineering fields like signal processing [26], electrical circuit design [16], scheduling [22], process controller evolution [25] and modelling of both steady-state and dynamic processes [21].

In this paper, optimal control of stochastic linear quadratic singular T–S fuzzy delay system is obtained using genetic programming. The linear T–S fuzzy system is the most popular fuzzy model due to its further intrinsic analysis: the linear matrix inequality (LMI)-based fuzzy controller is to minimize the upper bound of the performance index; structure oriented and switching fuzzy controllers are developed for more complicated systems [27]; the optimal fuzzy control technique is used to minimize the performance index from local-concept or global-concept approaches [32,33].

Stochastic linear quadratic regulator (LQR) problems have been studied by many researchers [1,5,6,12,31]. Chen et al. [11] have shown that the stochastic LQR problem is well posed if there are solutions to the Riccati equation and then an optimal feedback control can be obtained. For LQR problems, it is natural to study an associated Riccati equation. However, the existence and uniqueness of the solution of the Riccati equation in general, seem to be very difficult problems due to the presence of the complicated nonlinear term. Zhu and Li [37] used the iterative method for solving stochastic Riccati equations for stochastic LQR problems. There are several numerical methods to solve conventional Riccati equation as a result of the nonlinear process essential error accumulations may occur. In order to minimize the error, recently the conventional Riccati equation has been analyzed using neural network approach and genetic programming approach see [2–4,29]. A variety of numerical algorithms [9] have been developed for solving the algebraic Riccati equation.

Singular systems contain a mixture of algebraic and differential equations. In that sense, the algebraic equations represent the constraints to the solution of the differential part. These systems are also known as degenerate, differential algebraic, descriptor or semi state and generalized state space systems. The complex nature of singular system causes many difficulties in the analytical and numerical treatment of such systems, particularly when there is a need for their control. The system arises naturally as a linear approximation of system models or linear system models in many applications such as electrical networks, aircraft dynamics, neutral delay systems, chemical, thermal and diffusion processes, large scale systems, robotics, biology, etc., see [7,8,20]. As the theory of optimal control of linear systems with quadratic performance criteria is well developed, the results are most complete and close to use in many practical designing problems. The theory of the quadratic cost control problem has been treated as a more interesting problem and the optimal feedback with minimum cost control has been characterized by the solution of a Riccati equation. Da Prato and Ichikawa [13] showed that the optimal feedback control and the minimum cost are characterized by the solution of the Riccati equation. Solving the MRDE is the central issue in optimal control theory.

Although parallel algorithms can compute the solutions faster than sequential algorithms, there have been no report on genetic programming solutions [29] for MRDE. This paper focuses upon the implementation of genetic programming approach for solving MRDE in order to get the optimal solution. An example is given to illustrate the advantage of GP solution.

This paper is organized as follows. In Section 2, the statement of the problem is given. In Sections 3 and 4, solution of the MRDE is presented. In Section 5, numerical example is discussed. The final conclusion Section 6 demonstrates the efficiency of the method.

2. Statement of the problem

Consider the linear dynamical singular T–S fuzzy delay system [34] that can be expressed in the form:

R^i: If x_j is $T_{ji}(m_{ji}, \sigma_{ji})$, $i = 1, \ldots, r$ and $j = 1, \ldots, n$, then

$$F_i dx(t) = [A_i x(t) + B_i u(t-\tau)]dt + D_i u(t) dW(t), \quad x(0) = 0, \quad t \in [0, t_f], \tag{1}$$

where R^i denotes the ith rule of the fuzzy model, m_{ji} and σ_{ji} are the mean and standard deviation of the Gaussian membership function, the matrix F is possibly singular, $x(t) \in R^n$ is a generalized state space vector, $u(t) \in R^m$ is a control variable and it takes value in some Euclidean space, $W(t)$ is a Brownian motion and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $D \in \mathbb{R}^{n\times m}$ are known as coefficient matrices associated with $x(t)$ and $u(t)$ respectively, x_0 is given initial state vector and $m \leq n$.

By the following transformation [19],

$$y(t) = x(t) + \int_{-\tau}^{0} e^{-A(\eta+\tau)} Bu(t+\eta) d\eta,$$

the system dynamics (1) can be rewritten into a standard form of first order differential equation with out any explicit time delay term as

$$F_i dy(t) = [A_i y(t) + [B_i(A_i)]u(t)]dt + D_i u(t) dW(t), \quad y(0) = 0, \quad t \in [0, t_f], \tag{2}$$

where $B_i(A_i) = e^{-A_i\tau} B_i$, $B_i(A_i)$ is a $n \times m$ matrix.

In order to minimize both state and control signals of the feedback control system, a quadratic performance index is usually minimized:

$$J = E\left\{\frac{1}{2} y^T(t_f) F_i^T S F_i y(t_f) + \frac{1}{2}\int_0^{t_f} [y^T(t) Q y(t) + u^T(t) R u(t)] dt\right\},$$

where the superscript T denotes the transpose operator, $S \in \mathbb{R}^{n\times n}$ and $Q \in \mathbb{R}^{n\times n}$ are symmetric and positive definite (or semidefinite) weighting matrices for $x(t)$, $R \in \mathbb{R}^{m\times m}$ is a symmetric and positive definite weighting matrix for $u(t)$. It will be assumed that $|sF_i - A_i| \neq 0$ for some s. This assumption guarantees that any input $u(t)$ will generate one and only one state trajectory $x(t)$.