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ASOC-3890; No. of Pages 15
ARTICLE IN PRESS
Applied Soft Computing xxx (2016) xxx–xxx
Contents lists available at ScienceDirect
Applied Soft Computing
journal homepage: www.elsevier.com/locate/asoc

A new consistency concept for interval multiplicative preference relations

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ARTICLE INFO

Article history:
Received 1 March 2016
Received in revised form 6 June 2016
Accepted 1 November 2016
Available online xxx

Keywords:
Decision making
Interval multiplicative preference relation
Consistency analysis
0-1mixed programming model

ABSTRACT

Consistency analysis is very important to ensure the reasonable ranking order. However, all previous consistency concepts for interval multiplicative preference relations (IMPRs) are insufficient to address this type of preference relations. This paper introduces a new consistency concept for IMPRs that is a natural extension of the Saaty's consistency concept for multiplicative preference relations. Several desirable properties are discussed, and the relationship between the new concept and two previous ones is studied. Then, a method to judge the consistency of IMPRs is proposed. Considering the inconsistent case, a 0–1 mixed programming model to derive consistent IMPRs from inconsistent ones is established. To determine missing values in incomplete HFPRs, 0–1 mixed programming models are constructed that can address the situation where ignore objects exist. Meanwhile, illustrative examples are offered to show the feasibility and efficiency of the developed theoretical results, and comparison analysis is provided. Finally, a consistency analysis based algorithm to decision making with IMPRs is developed that can address inconsistent and incomplete cases.

1. Introduction

The analytic hierarchy process (AHP) has been widely used in decision making, which is a powerful tool to fully rank compared objects. Since it was first proposed by Saaty [1], the AHP has been largely developed [2–11]. In traditional preference relations, decision makers need to give precise judgments. This limits the application of the AHP because it is more and more difficult or impossible to require the decision makers to give precise preferences on compared objects. To solve this issue, Saaty and Vargas [12] introduced the concept of IMPRs to address uncertainties of the decision makers. Meanwhile, the authors developed a Monte Carlo simulation approach to obtain the interval priority weight vector. After the pioneer work of Saaty and Vargas [12], many methods for decision making with IMPRs are developed. For example, Arbel and Vargas [13] treated interval bounds as hard constraints and explored two approaches to generate the priority weight vector when preferences are expressed as intervals: a simulation method and a mathematical programming method. Similar to the method of Arbel and Vargas [13], Xia and Xu [14] constructed a goal programming model to derive the interval priority weight vector by considering interval bounds as constraints. Using the linear programming model, Chandran et al. [15] presented an approach to obtain the interval priority weight vector from IMPRs. Because this method is given into two stages, it is also called the two-stage logarithmic goal programming method. Islam et al. [16] developed a lexicographic goal programming (LGP) method to determine the priority weights from inconsistent IMPRs and explored several properties and advantages of LGP method. However, this method is defective in theory [17]. More researches about IMPRs can be seen in Refs. [18–21].

The main issue of above mentioned researches is their failure to handle inconsistent case. To address inconsistent IMPRs, Wei et al. [22] gave a consistency concept for IMPRs, which is a direct application of the Saaty's consistency concept. However, this concept is incorrect because many properties of operations on real numbers no longer hold on intervals. Wang et al. [17,23] considered that an IMPR is consistent when there is a consistent multiplicative preference relation. This consistency concept has several disadvantages too. For instance, the feasible region might be empty for an IMPR with acceptable consistency [24]. Later, Liu [24] presented a consistency concept for IMPRs by

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http://dx.doi.org/10.1016/j.asoc.2016.11.001
1568-4946/

Please cite this article in press as: F. Meng, C. Tan, A new consistency concept for interval multiplicative preference relations, Appl. Soft Comput. J. (2016), http://dx.doi.org/10.1016/j.asoc.2016.11.001

considering two associated crisp multiplicative preference relations. Based on this consistency concept, the author developed a method to derive the interval priority weight vector. Later, Liu et al. [25] used Liu's consistency concept [24] to develop a method to estimate missing values in an incomplete IMPR. Recently, Wang [26] noted that this consistency concept suffers from serious defects, which is technical wrong. The ranking order might be different for an identical IMPR only because of different compared orders of objects. Then, Wang [26] gave a new consistency concept for IMPRs, Nevertheless, Wang's consistency concept is insufficient to address inconsistent/incomplete IMPRs, different ranking orders might be derived with respect to the same inconsistent/incomplete IMPR. This makes that Wang's method is insufficient. As we know, consistency analysis is very important to ensure a reasonable ranking order [4,5,9,22,24,26–29]. However, all consistency concepts for IMPRs are insufficient to address this type of preference relations. This limits its application in practice.

Consistency analysis is a necessary step in decision making with preference relations. To address the limitations in the previous consistency concepts for IMPRs, this paper defines a new consistency concept. Then, several desirable properties of this concept are discussed, by which one can find:

i. It overcomes the limitations in the previous consistency concepts.
ii. It considers the consistency of intervals' left and right endpoints simultaneously.
iii. It is a natural extension of the Saaty's consistency concept for the crisp case.
iv. It is robust to the labels of objects.

Furthermore, the relationship between the new concept and that given in Refs. [24,26] is studied. Subsequently, 0–1 mixed programming models to derive consistent IMPRs and to determine missing values in incomplete IMPRs are constructed. Meanwhile, numerical examples are provided to show the concrete application of constructed models, and comparison analysis is made. Finally, an algorithm to decision making with IMPRs is developed that can address the inconsistent and incomplete cases.

The rest part of this paper is organized as follows: Section 2 first reviews the concept of IMPRs and four consistency concepts. Meanwhile, we analyze their limitations from theoretical and numerical aspects, which show that previous consistency concepts are insufficient to address this type of preference relations. Section 3 presents a new consistency concept for IMPRs, which is a natural extension of the crisp case and independent of the object labels. Then, the relationship between the new consistency concept, Liu's consistency concept and Wang's consistency concept is studied. Furthermore, a method to judge the consistency of IMPRs is introduced. Section 4 constructs a 0–1 mixed programming model to derive consistent IMPRs from inconsistent ones, which is based on the new consistency concept. Section 5 constructs two 0–1 mixed programming models to determine missing values in an incomplete IMPR, which have the highest consistent level with respect to the known interval judgments. The conclusions and remarks are provided in the last section.

2. Several basic concepts

For simplicity, let $X=\{x_1, x_2, \ldots, x_n\}$denote the set of compared objects. To express the decision makers' uncertain preferences, Saaty and Vargas [12] introduced the concept of IMPRs.

Definition 1. ([12]) An IMPR B is defined as follows:

$$B=(b_{ij})_{n\times n}=\begin{pmatrix} [1,1] & [b_{12}^-, b_{12}^+] & \ldots & [b_{1n}^-, b_{1n}^+] \\ [b_{21}^-, b_{21}^+] & [1,1] & \ldots & [b_{2n}^-, b_{2n}^+] \\ \vdots & \vdots & \vdots & \vdots \\ [b_{n1}^-, b_{n1}^+] & [b_{n2}^-, b_{n2}^+] & \ldots & [1,1] \end{pmatrix},$$

where $b_{ij}^-, b_{ij}^+ > 0$ such that $b_{ij}^- \le b_{ij}^+$, $b_{ij}^- = 1/b_{ji}^+$ and $b_{ij}^+ = 1/b_{ji}^-$, b_{ij} indicates that x_i is between $b_{ij}^-$and b_{ij}^+ times as important as x_j. When $b_{ij}^- = b_{ij}^+$ for all $i, j = 1, 2, \ldots, n$, B degenerates to a multiplicative preference relation [1], namely, $A = (a_{ij})_{n\times n}$ with $a_{ij} > 0$, $a_{ij} = 1/a_{ji}$, where $a_{ij} = b_{ij}^- = b_{ij}^+$ for all $i, j = 1, 2, \ldots, n$.

With respect to IMPRs, there are four main consistency concepts:

(i) Let $B = (b_{ij})_{n\times n}$ be an IMPR. If $b_{ij} = b_{ik} \otimes b_{kj}$ for all $i, k, j = 1, 2, \ldots, n$, then B is said to be consistent [22], where $\otimes$ is interval multiplication operation. From Minkowski operations on intervals, one can easily see that this consistency concept is not true, namely, $b_{ij} = b_{ik} \otimes b_{kj}$ does not hold for all $i, k, j=1, 2, \ldots, n$.

(ii) Given an IMPR $B=(b_{ij})_{n\times n}$, if the following convex feasible region

$$S_w = \left\{ w = (w_1, w_2, \ldots, w_n) | b_{ij}^- \le w_i/w_j \le b_{ij}^+, \sum_{i=1}^{n} w_i = 1, w_i > 0 \right\}$$

is nonempty, then B is said to be consistent [17,23]. However, this consistency concept has limitation. For instance, in some times, it may be inevitable to adjust the range of intervals [24].