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ASOC-3864; No. of Pages 17

ARTICLE IN PRESS

Applied Soft Computing xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Memetic algorithms for reconstruction of binary images on triangular grids with 3 and 6 projections

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ARTICLE INFO

Article history:
Received 11 December 2015
Received in revised form 15 August 2016
Accepted 10 October 2016
Available online xxx

Keywords:
Triangular grid
Image reconstruction
Memetic algorithm
Binary tomography
Genetic algorithm
Non-traditional grid

ABSTRACT

When an image is given with only some measurable data, e.g., projections, the most important task is to reconstruct it, i.e., to find an image that provides the measured data. These tomographic problems are frequently used in the theory and applications of image processing. In this paper, memetic algorithms are investigated on triangular grids for the reconstruction of binary images using their three and six direction projections. The algorithm generates an initial population using the network flow algorithm for two of the input projections. The reconstructed images evolve towards an optimal solution or close to the optimal solution, by using crossover operators and guided mutation operators. The quality of the images is improved by using switching components and compactness operator.

1. Introduction

Tomography is one of the main fields of image processing [1,2]. In some optimization problems the task is to estimate/compute a set of data from some known connected data. Tomography deals with reconstruction of images from their projections. The measured, known data are the projection values for various (usually, few) directions; the set of data to compute is the original image. Mathematically, the image corresponds to a function and the problem is to reconstruct this function from its integrals or sums over subsets of its domain. In general, the tomography problem can be continuous or discrete. In discrete tomography the domain of the function is discrete, and the range of the function is a finite set of real numbers, usually nonnegative numbers. The simplest case of discrete tomography is the binary tomography: it deals with the problem of the reconstruction of a binary image from a small number of projections. A binary image contains only black and white pixels and it is represented by a function having values only 0's and 1's, respectively.

Tomography is used in several fields in practice, when data about the inner structure of the object are needed without breaking it. Radiation sources and sensors are used outside of the object and the measured data are used to reconstruct or predict the shape of the object. It is done by computers using various algorithmic approaches. These techniques are applied in metal industry, and, for instance, in medical applications: e.g., Positron Emission Tomography (PET), Computer Tomography (CT) [1–3].

We note here that binary tomography (on the square grid) with two projection directions is undetermined, i.e., usually there are a large (even an exponential) number of solutions exist [4]. In the case of more than two projections uniqueness, consistency and reconstruction problems are in general NP-hard [5].

The first studied grid for discrete tomography was the square grid [4,6–9] and then, the cubic grid, because the Cartesian coordinate system fits very well to these grids. However, other regular, but non-traditional grids are also used in image processing. In the plane, considering the possible tiling, the hexagonal and triangular grids can be used instead of the square grid. The advantage of the hexagonal grid is that it is very simple having only one neighborhood relation and it has better symmetric properties than the square grid has. The symmetry of the hexagonal grid is mirrored by its description with zero-sum coordinate triplets [10]. There is a relatively wide literature on binary tomography using the hexagonal grid [11–13]. The triangular grid has similar symmetric properties as the hexagonal grid, a rotation by $\pi/3$ or by $2\pi/3$ (depending on the center of the rotation) moves the grid to itself; a description by three coordinates can effectively be used [14,15] (see also

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http://dx.doi.org/10.1016/j.asoc.2016.10.014
1568-4946/

Please cite this article in press as: B. Nagy, E.V. Moisi, Memetic algorithms for reconstruction of binary images on triangular grids with 3 and 6 projections, Appl. Soft Comput. J. (2016), http://dx.doi.org/10.1016/j.asoc.2016.10.014

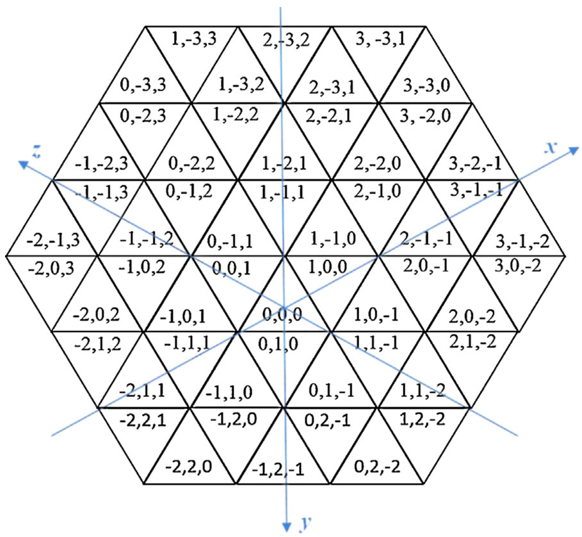

Fig. 1. Coordinate representation of a hexagon-shaped image of size $n = 6$.

Fig. 1). We note here that all these three regular girds can be found in the nature connected to meshes of various crystal structures [16]. Apart from the good symmetric properties, the advantage of the triangular grid could be the flexibility caused by the three usual neighborhood [17–19] which allows a wider choice of image processing operations in various applications.

Genetic algorithms are evolutionary algorithms in which agents represent various proposed solutions of the state space and their community develops by mimicking the Darwinian evolution [20]. Roughly speaking, memetic algorithms are those types of genetic algorithms which are also equipped some local search algorithms, e.g., individual learning, hill climbing by local search heuristics or other local search strategies [21]. We believe that we do not need to detail the theory and history of genetic and memetic algorithms to the readership of the journal, otherwise they are referred to Part III of [22].

Genetic approach is widely used in image processing [23,24], and, especially, in connected medical applications [25–27]. They are also frequently used in optimization problems providing relatively good, reasonable solutions in a relatively short time [28,29]. There are several examples also where genetic approaches are used in various tomographic problems, e.g., [30,31].

With projections in three directions, usually, the vertical and horizontal projections are complemented by projections from one of the diagonal directions. However, this scenario, even it has some mathematical interest, is not realistic. Projections according to three equally distributed directions look more realistic. Images on the triangular grid allow a natural possibility to this scenario.

Based on the previous parts we can state that the tomography problems are very important for various industrial and medical applications. Since the triangular grid gives some new features due to its symmetry, it is interesting both from theoretical point of view and also from the practical side what we can say about and how we can solve such problems. In this paper we address both theoretical aspects (by, e.g., describing switching components) and practical results (by giving algorithms with experiments).

The contents of the paper is as follows. In the next sections, we give two memetic algorithms to solve the tomography problem of the reconstruction of a binary image represented on a triangular grid. Our memetic algorithms are somewhat analogous to the method given in [32] and in [33,34]. Also, the so-called switching components and a compactness operator are described and used here. A switching component consists of a set of points such that inverting their values the projection data do not change. A compactness operator eliminates some of the isolated points of an image. It also changes some of the projection values. In the case of the triangular grid we use data from three or six projections for the reconstruction of images. The used directions are the natural directions of the triangular grid, based on the symmetry of the grid and thus, they can easily be described by the symmetric coordinate frame. The first three natural directions are using projections that gather the information from pixels sharing a coordinate value (projections by lanes, i.e., directions orthogonal to one of the coordinate axes). Mathematical descriptions of switching components that we have found are also provided. When we use six directions we use three additional directions that are parallel to the symmetry axes, they are the diamond-chains (see also Fig. 2). We note here that in our earlier works [35,36] we have presented memetic and evolutionary algorithms, respectively, with various genetic operators using three projection directions by lanes. Now, in this paper, we complete our task by presenting a more detailed description and an extension of the memetic algorithm using six projection directions similarly as six directions were also used in [37] (where a simulated annealing technique was used to solve similar problems). Experimental results validate our approach.

2. Images on the triangular grid

We will introduce some basic notations and some definitions regarding the discrete tomography problem on the triangular grid. The theoretical parts about discrete tomography are based on the theoretical results known for the square grid (see, e.g., [1,4,32]). We start the description by a brief overview on the triangular grid.

For the sake of completeness we recall how to assign the coordinate triplets to the pixels (also referred as points) from, e.g., [38]. We choose a pixel to be the origin and assign coordinate values (0, 0, 0) to it. We take the three lines through the center of this triangle, which are orthogonal to its sides and fix these lines as the coordinate axes x, y and z. The coordinate values are assigned to the pixels inductively. Let the coordinate values of a triangle A be known. Consider a triangle B, which has not coordinate values yet and has a common side with A. This common side is orthogonal to one of the coordinate axes. According to the direction of this axis, the corresponding coordinate value of A is increased or decreased by 1 to get the corresponding coordinate of B. The other two values of A and B are equal.

In this paper, we will assume that all our images are represented as binary hexagon-shaped sets of size $n \times n \times n$ (i.e., regular hexagons), where n is the double of the length of the side of the regular hexagon. The number of pixels in such a hexagon-shaped set is $3n^2/2$. As we use the triangular grid for the representation of our images, we have two types of pixels, i.e., there are two orientations, also called parities (see, e.g., [15]) of the triangles, thus each pixel is even ($\triangle$) or odd ($\triangledown$). Since the grid is a 2D one, the coordinate values are not independent of each other, but each triangle of the shape $\triangle$ is addressed with a triplet having sum 0, and each triangle of the shape $\triangledown$ is addressed with a triplet with sum 1 [14,36,38]. Fig. 1 shows the coordinate values of the pixels of a hexagon-shaped image of size $n = 6$.

The problem considered is the reconstruction of binary hexagon-shaped set from its projections orthogonal and parallel to the three axes of the grid (see Fig. 1). In these images the pixels have values black (0) or white (1).