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Observer-based method for synchronization of uncertain fractional order chaotic systems by the use of a general type-2 fuzzy system

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ABSTRACT

Synchronization of the fractional order chaotic systems is extensively studied in recent years due to its potential applications in many branches of science and engineering. The main problems in this field are that the dynamics of the system in hand are often uncertain and are perturbed by external disturbances. Also the unknown nonlinear functions in the system dynamics are generally complicated and in many practical applications we have measurement errors and unavailable states. In this paper, a novel robust and asymptotically stable controller is proposed to synchronize uncertain fractional order chaotic systems. Its design is based on linear matrix inequality (LMI) technique. Furthermore, an observer is presented to estimate the unavailable states. A general type-2 fuzzy system (GT2FS) based on α-plane representation with Gaussian secondary membership functions (MF) and type-2 non-singleton fuzzification is proposed to approximate the unknown complex nonlinear functions in the dynamics of system. The input uncertainties associated with the observer error and the malfunctioning of the input devices are modeled by interval type-2 fuzzy MFs instead of crisp numbers. To decrease the computational cost of the GT2FS, a simple type-reduction method is proposed. The antecedent parameters of GT2FS are tuned based on a modified form of social spider optimization (SSO) algorithm. The simulation examples show that the proposed control scheme gives high performance in the presence of unknown functions, external disturbances and unavailable states. The performance of GT2FS with different α-levels and different fuzzification methods are compared with type-1 and interval type-2 fuzzy systems in several examples.

1. Introduction

Synchronization of chaotic systems is among the most widely researched topics in recent years, because of its potential applications in many branches of science and engineering such as secure communications, information processing, chemical science, biological systems (see [1] for a review). The main problem in this field is that the mathematical model of the system is unknown. In many practical applications it is difficult to determine the exact model of the system. Furthermore, because of the failure of the physical devices and the external disturbances, the mathematical model of the system is subject to change. This is true especially in the synchronization of chaotic systems, in which the controlled system is likely to be subjected to disturbances and parameter perturbations. The robustness of the synchronization then becomes very important. Also the unknown nonlinear functions in the dynamics of the system are more complicated and a strong tool is need to handle the high level of uncertainty. Furthermore, in practical applications, especially in the field of synchronization of chaotic systems, it is difficult to measure all the states of the system. Additionally errors in the measurement are a common problem.

It is well known that fuzzy systems can evaluate the uncertainties and can uniformly approximate any non-linear continuous function over a compact set [2]. By using this property of the fuzzy systems, fuzzy controllers have often been used for the synchronization of chaotic systems, as they do not need an accurate model of system. Furthermore, fuzzy controllers can handle human expert knowledge.

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For instance the adaptive type-1 fuzzy sliding mode control and interval type-2 fuzzy sliding mode control are presented in [3,4]. In these papers, the fuzzy systems are used to estimate unknown nonlinearities. In [5], a class of fractional order chaotic systems is described by Takagi–Sugeno fuzzy model, and then a fuzzy state feedback controller is designed. In [6] an adaptive fuzzy logic system is used for the estimation of the unknown functions and then the generalized projective synchronization problem for the incommensurate fractional order chaotic systems is solved.

Many methods have been presented for the stability and robustness analysis of fuzzy controllers. For instance in [7], the multivariable nonlinear systems are represented by the Takagi–Sugeno fuzzy model and then based on a single-grid-point approach, stability and robustness conditions are derived. In [8], the stability analysis of Takagi–Sugeno fuzzy controllers is done based on Lyapunov's direct method. In [9], by using Lyapunov's stability theory and the linear matrix inequality (LMI) technique, sufficient conditions are derived for proving the local stability. The problems of the robust stabilization for fractional order systems based on the fuzzy models has rarely been investigated. For instance in [10], in terms of LMI technique, the sufficient and necessary condition of asymptotical stability for fractional order Takagi–Sugeno fuzzy model is derived and is applied on the fractional order Van der Pol system. In [11], an adaptive adjustment mechanism by using Takagi–Sugeno fuzzy systems is presented for linear fractional order systems.

It is well known that interval type-2 fuzzy systems (IT2FSs) can perform better than type-1 fuzzy systems (T1FSs) in the presence of noise and other uncertainties, and general type-2 fuzzy systems (GT2FSs) can result in an even better performance than IT2FSs [12]. Briefly, the membership degree of a type-1 fuzzy set is a crisp value while in the type-2 fuzzy set is a fuzzy number. The secondary membership function of the interval type-2 fuzzy set is equal to one, whereas in a general type-2 fuzzy set, is a type-1 fuzzy set itself. This property causes that a high level of uncertainties can be modeled by the type-2 fuzzy sets. Although T1FSs and IT2FSs have widely been studied, the applications of GT2FSs are relatively new, only a limited number of applications can be seen in literature. For instance in [13], a PI controller is designed by using GT2FSs. In [14] a GT2FS is used in emotion recognition problem. An edge detection method based on GT2FSs is presented in [15]. A direct adaptive general type-2 fuzzy controller is designed in [16]. A pattern recognition technique based on the general type-2 fuzzy sets is proposed in [17]. A generalized type-2 fuzzy controller is designed in [18] for a mobile robot and it is shown that GT2FSs can better handle uncertainty. One of the important limitation of GT2FSs is the high computational cost. Most recently, some simplifications have been made on GT2FSs by using the vertical-slice and horizontal representation of the general type-2 fuzzy sets [12,19]. However, for real-time practical applications GT2FSs, additional research work is needed.

Recently, population based algorithms are frequently used to optimize fuzzy controllers. Because they are able to provide good results in a reasonable cost and time [20]. For instance in [21,22], bio-inspired algorithm based on the bee behavior is presented for optimizing fuzzy controllers. In [23] fuzzy and ant colony optimization based method is presented for wireless sensor networks. The genetic and particle swarm optimization algorithms are used in [24,25], to optimize type-1 fuzzy based controllers. A review of the bio-inspired methods used in the interval type-2 fuzzy controllers is considered in [20]. In [26] a hybrid learning algorithm based on particle swarm optimization and recursive least squares algorithm is proposed for learning Takagi–Sugeno–Kang type fuzzy neural network which uses general type-2 fuzzy sets in a type-2 fuzzy logic system. Too few number of papers present appropriate methods to optimize the general type-2 fuzzy systems. The social spider optimization (SSO) algorithm is recently proposed by Erik and et al. [27] and it is shown by simulation studies on several benchmark examples that SSO algorithm results in a better performance in contrast to classical swarm optimization and artificial bee colony algorithms. In this paper, a modified SSO algorithm is presented to optimize the proposed GT2FS.

In the above-mentioned fuzzy controllers, singleton fuzzification is used, i.e. crisp numbers models the inputs of the fuzzy system. Furthermore it is assumed that the states of the system are available. In the referenced works, the robustness of the controller against approximation errors and external disturbances is not completely investigated. Also few works have done in the design and optimization of general type-2 fuzzy systems. The contributions of this paper are summarized as follows:

- An observer is designed to estimate the unknown states.
- A general type-2 fuzzy system with Gaussian secondary membership function and non-singleton fuzzification and simple type-reduction is proposed.
- A modified social spider optimization algorithm is presented to optimize proposed GT2FS.
- The effects of approximation error and external disturbances are eliminated by the proposed robust LMI based controller.
- The effectiveness of the proposed GT2FS and controller is shown by several simulation examples.

The reminder of this paper is organized as follows: Problem formulation and some preliminaries are given in Section 2. The proposed general non-singleton type-2 fuzzy system is presented in Section 3. The SSO algorithm is illustrated in Section 4. The stability analysis is given in Section 5. The simulations results are provided in Section 6. Finally the main results obtained are summarized in Section 7.

2. Problem formulation and preliminaries

2.1. preliminaries

Let f be a continuous function on $\mathbb{R}^+$ and $q > 0$, then fractional integral and the fractional order derivative in the sense of Riemann–Liouville, are defined as follows:

$$I_t^q = \frac{1}{\Gamma(q)} \int_0^t (t-\tau)^{q-1} f(\tau) d\tau \tag{1}$$

$$D_t^q f(t) = \frac{d^m}{dt^m} \left[\frac{1}{\Gamma(m-q)} \int_0^t \frac{f(\tau)}{(t-\tau)^{q-m+1}} d\tau \right] \tag{2}$$

where $\Gamma(\cdot)$ is Gamma function, m is an integer so that $m-1 < q < m$.