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A general higher-order one-dimensional model for large deformation analysis of solid bodies

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Highlights

- A general higher-order theory for one-dimensional analysis of 3-D structures is presented.
- A very general approximation of the displacement field of the cross-section of the structure is used.
- A weak-form displacement finite element model that is suitable for 3-D analysis is developed.
- The present formulation is very general but it is a reduced-order model which is capable of modeling shell structures.
- Validated the model using some benchmark problems against shell finite elements.

Abstract

In the present study, a general higher-order one-dimensional model for large deformation analysis of 3-D solids is developed. The displacement vector of the cross-section or slices of a 3-D body are approximated in the reference frame using general basis functions in polar coordinates. Using the principle of virtual displacements, we obtain the governing equations of motion for large deformation. Further, we develop weak-form finite element model of the theory. The finite element model is used to obtain solutions for cylindrical shells under internal pressure and pinching point forces. The solution obtained for the cylindrical shell under internal pressure from this one-dimensional analysis have been compared with the solutions obtained with the 7-parameter shell theory of Reddy and his colleagues. The results are found to be in good agreement although a one-dimensional model is used.

Keywords: General higher-order theory; One-dimensional theory; Analysis of 3-D body; Analysis of shells; Nonlinear finite element analysis; Large deformation

1. Introduction

In this study, we develop a general higher-order one-dimensional model to analyze three-dimensional bodies. All three-dimensional structures are often analyzed by 3-D elasticity or equivalent 2-D shell theories to obtain the response due to various loads. Reductions of dimensionality of problems are very common in solid mechanics. For example,

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various beam theories convert the 3-D problem of slender bodies with certain set of loads into one-dimensional problem by approximating the displacement field in such a way that the unknowns introduced are only functions of one coordinate (see Reddy [1]). In this case, the geometry should be such that the width and height of the solid are very small compared to the length of the solid. Similarly, various plate and shell theories (see Reddy [2]) are formulated to convert 3-D problems into two-dimensional problems by approximating the displacement field in terms of variables in the mid-surface of the plate or shell (see [3–9] for higher-order shell theories). The geometry should be such that the thickness be small compared to the other two dimensions in order the assumed displacement field be a good approximation. In the present study, we generalize this reduction in the dimension of the problem to one even for plate and shell like geometries. This is done by approximating the displacement field of the cross-section or slices of a 3-D body by considering general basis functions (e.g., Fourier series or other similar series of polynomial basis functions) in polar coordinates in the plane of the cross-section. Truncated Fourier or other appropriate polynomial series would be good enough for approximating the displacement field of the cross-section. Based on such general displacement approximation, we can use the principle of virtual displacements to obtain the governing differential equations of 3-D body in terms of the coefficients of the basis functions used in the approximation. The formulation is general enough to analyze large deformations of plates and shells.

The present formulation of converting three-dimensional analysis to one-dimensional analysis and its finite element model have not been reported in present solid mechanics literature, which is the major contribution of this study. Solid mechanics problem of three- or two-dimensional bodies can be analyzed by three- or two-dimensional finite element model, but one-dimensional analysis of such problem is certainly not reported. The one-dimensional finite element model allows higher-order continuity element (C^n, $n > 0$ continuity) or any order with any number of nodes with general Hermite interpolation functions, which is not possible in the case of two- or three-dimensional domains. Problems with higher-order continuity requirement of the unknown variables arises very often in Cosserat continuum and other nonlocal continuum theories; for example, rotation gradient dependent theory for Cosserat continua (see Srinivasa and Reddy [10] and Arbind, Reddy and Srinivasa [11,12]) require C^1 continuity of unknown variables in the case of beam, plate, and shell structures. Other higher-order strain gradient-dependent theories (see Khodabakhshi and Reddy [13]) require C^2 or higher-order continuity of unknown variables. In such cases, reduction of 2-D or 3-D problem to one-dimensional analysis would be very useful as far as finite element modeling is concerned.

Using the present higher-order theory, a finite element model for large deformation analysis of structures is developed. This model can be employed to analyze various shell structures; for example, cylindrical shells with constant or varying radius of curvature or structures with solid arbitrary cross-sections can be analyzed. Other applications could be modeling of straight ducts or beams with arbitrary cross sections under a system of body or surface forces in three dimensions, when the cross-section of the beam or duct deforms in three dimensions. The existing 2-D or 3-D beam theories would not be able to model such phenomenon. Only fixed cross-section or centroid of the cross-section can be considered directly as boundary conditions as the presented one-dimensional theory approximate the displacement components of the entire cross-section. But any general boundary condition can be included through the constrained condition in the finite element formulation, which is also presented in this study. In the numerical results section, the capabilities of the model are illustrated using a cylindrical shell structure under internal pressure and the results are also compared with that of shell theory obtain in Rivera and Reddy [14] as a benchmark problem. Numerical results are also presented for pinching of a cylindrical shell and semi-cylindrical shell subjected to point load in the case of large deformation and the solutions data are compared with that of 7-parameter shell theory and ANSYS.

2. Governing equations of motion

Let us consider the cylindrical coordinate system (x, r, θ) in the reference frame of a solid body, whose longitudinal axis coincides with the x-axis and the polar coordinates (r, θ) define the cross-section of the body (which is perpendicular to the x-axis). The solid body is acted upon a system of body and surface forces that deform the body and reach the equilibrium. The displacement field at a point during of motion in the assumed coordinate system is expressed as

$$\mathbf{u} = u_x \hat{\mathbf{e}}_x + u_r \hat{\mathbf{e}}_r + u_\theta \hat{\mathbf{e}}_\theta \tag{1}$$