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Multi-level Bézier extraction for hierarchical local refinement of Isogeometric Analysis

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Highlights

- We present a way to implement hierarchically refined Isogeometric Analysis.
- Our main focus is on (Truncated) Hierarchical B-Splines and Isogeometric Analysis.
- We present a concept applicable to different kinds of refinements and functions.
- The proposed local approach is suitable for parallel and non-linear simulations.
- Basic algorithms for knot insertion on spline spaces have been outlined and compared.

Abstract

One of the main topics of research on Isogeometric Analysis is local refinement. Among the various techniques currently studied and developed, one of the most appealing, referred to as hierarchical B-Splines, consists of defining a suitable set of basis functions on different hierarchical levels. This strategy can also be improved, for example to recover partition of unity, resorting to a truncation operation, giving rise to the so-called truncated hierarchical B-Splines. Despite its conceptual simplicity, implementing the hierarchical definition of shape functions into an existing code can be rather involved. In this work we present a simple way to bring the hierarchical isogeometric concept closer to a standard finite element formulation. Practically speaking, the hierarchy of functions and knot spans is flattened into a sequence of elements being equipped with a standard single-level basis. In fact, the proposed multi-level extraction is a generalization of the classical Bézier extraction and analogously offers a standard element structure to the hierarchical overlay of functions. Moreover, this approach is suitable for an extension to non-linear problems and for a parallel implementation. The multi-level extraction is presented as a general concept that can be applied to different kinds of refinements and basis functions. Finally, few basic algorithms to compute the local multi-level extraction operator for knot insertion on spline spaces are outlined and compared, and some numerical examples are presented.

Keywords: Isogeometric Analysis; Local refinement; (Truncated) hierarchical B-splines; Bézier extraction

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1. Introduction

Isogeometric Analysis (IGA) [1,2] is a powerful approach for the numerical analysis of problems governed by partial differential equations (PDEs). Its aim is to bridge the gap between geometry and simulation by performing finite element analysis using the same functions that represent the geometry in Computer Aided Design (CAD). Classically, these include tensor product B-Splines and Non-Uniform Rational B-Splines (NURBS). Thanks to the high-regularity of the adopted basis functions, IGA has shown better accuracy per-degree-of-freedom with respect to standard finite elements in many applications ranging from solids and structures (cf., e.g., [3–6]) to fluids (cf., e.g., [7,8]) and fluid–structure interaction (cf., e.g., [9,10]), opening also the door to geometrically flexible discretizations of higher-order PDEs in primal form (cf., e.g., [11,12]).

Within this context, adaptivity has become a fundamental topic of research, as the tensor product structure of B-Splines and NURBS precludes local refinement. Therefore, different techniques are being currently developed, including T-Splines (see, e.g., [13,14]), LR-Splines [15] and hierarchical B-Splines ($\mathcal{HB}$) (see, e.g., [16–18]). $\mathcal{HB}$ are based on the definition of a suitable set of basis functions on different hierarchical levels and look like one of the most promising ways to effectively implement local refinement in IGA. This strategy has been recently improved under the name of truncated hierarchical B-spline ($\mathcal{THB}$) [19,20], with the aim to recover partition of unity and improve the bandwidth granted by standard $\mathcal{HB}$. Following its mathematical formulation, these techniques can be implemented with ad-hoc algorithms (see, e.g., [21]). However, despite its conceptual simplicity, implementing the hierarchical definition of shape functions into an existing code can be rather involved.

In this work, we present a simple way to bring the hierarchical concept closer to a standard finite element formulation. From the practical point of view, the hierarchy of functions and knot spans is locally flattened into a sequence of elements being equipped with a standard single-level basis. In fact, the proposed multi-level extraction is a generalization of the classical Bézier extraction [22] and analogously offers a standard element structure to the hierarchical overlay of functions. The proposed approach is suitable for an extension to non-linear problems and for a parallel implementation. In addition, the multi-level extraction is presented as a general concept that can be applied to different kinds of refinements and basis functions.

In the literature, several works moving along analogous research lines can be found. In [23] a similar approach is applied globally level-wise, where the active entries of the system matrices are computed in a standard way for each level. Then, each level system matrix is transformed and assembled to the hierarchical system. One advantage of this approach is that no connectivity information is needed. However, for large and parallel applications a local method is more efficient. Moreover, for non-linearities, the solution or its derivatives have to be evaluated at each integration point. This is naturally a local operation that would be less efficient when following a global strategy. Another similar global approach can be found in [24], where the active entries of the system matrices K^l are computed for each level l, they are inserted in the diagonal of a diagonal block matrix K and, finally, K is transformed to the hierarchical system. The advantages and drawbacks are similar to the ones discussed for [23]. In [24,25], Bézier extraction is applied level-wise in a standard fashion, while we propose to apply it just once per element, combined with the multi-level extraction. This way we extract at once all the hierarchical functions supporting the element. A very similar framework is presented in [26,27], but it is restricted to the overlay of uniform knot vectors obtained by bisection, no knot repetition (in particular, no open-knot vector is permitted) and the approach does not extend to truncated hierarchies. Moreover, the method in [26] is developed with the restriction of [18], i.e. refinements on the boundary are not allowed. Here, we consider more general refinements, truncated hierarchies, knot repetition, open-knot vectors and we provide alternative algorithms to produce the extraction operators. Another closely related concept is outlined in [28–30], but also here truncation is not considered and we conceptually separate the multi-level extraction from the standard Bézier extraction, setting a more general independent framework applicable to any sequence of nested space. In particular, we consider possibly different target hierarchical spaces, i.e., $\mathcal{THB}$, and different standard functions, like B-splines, Bernstein or Lagrange polynomials. In addition, we present and compare different algorithms to compute the multi-level extraction operator for knot insertion on spline spaces.

The structure of the paper is as follows. Section 2 introduces the $\mathcal{HB}$ and $\mathcal{THB}$ refinement strategies together with associated fundamental concepts. Section 3 discusses the multi-level extraction and its basic algorithms. Finally, Section 4 presents some numerical experiments.

2. Preliminaries

In this section, we concisely discuss some preliminary concepts we will use throughout the paper. In particular, we present B-Splines and NURBS, the idea of Bézier extraction, as well as hierarchical splines.