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Smoothing gradient damage model with evolving anisotropic nonlocal interactions tailored to low-order finite elements

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Highlights

- We present a smoothing gradient damage model for localized failure in quasi-brittle materials.
- Our proposed damage model is tailored to low-order finite elements.
- We introduce a modified evolving anisotropic nonlocal gradient parameter to eliminate spurious damage growth.
- We control the softening behavior through fracture energy and characteristic length.
- We propose a novel bi-energy norm based equivalent strain to shear band problem.

Abstract

This paper presents a novel smoothing gradient damage model, which goes beyond certain limitations of conventional methods, for accurate prediction of localized failure in quasi-brittle materials. The proposed method is particularly tailored to low-order finite elements such as 4-node quadrilateral or 3-node triangular elements. The low-order elements are preferable in practice as they can automatically be generated for problems even with complex geometries at low computational cost. In order to eliminate spurious damage growth and correct wrong prediction of shear band induced by using the *constant* gradient parameter in terms of conventional models, we thus introduce a novel modified *evolving* anisotropic nonlocal gradient parameter, which aims to control the behavior of nonlocal interactions of damage microprocess during the entire loading history in a more appropriate manner. Unlike conventional approaches, the novel modified evolving gradient parameter heavily depends on the principal stress and equivalent strain states, which serves to reduce the impact of localized deformation. The stress fields thus play a crucial role in the present formulation as they greatly affect the orientation and intensity of nonlocal interactions. The quality of raw stresses becomes critical (e.g., stress oscillation) once two unknowns (i.e., displacements and nonlocal equivalent strain) are approximated simultaneously using the same orders of interpolation functions (e.g., linear–linear). A smoothing technique is thus adopted to *smooth out* the raw stresses. The stresses after smoothing are shown adequately in the estimation of the new gradient parameter, providing much better solutions. In addition, to precisely capture the softening in quasi-brittle materials, the original energy norm is decomposed into tensile and compressive parts to form a new bi-energy norm. The scalar equivalent strain is thus estimated through this new bi-energy norm, which is obviously able to distinguish tensile and compressive conditions. To further enhance the capability of the present damage model, the material softening process is also determined through fracture energy in terms of fracture mechanics. Comparison of the present results with reference solutions derived from experimental data and

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other numerical methods for benchmark applications, regarding the nonlocal equivalent strain, damage profile, structural force–displacement curves, etc., confirms the accuracy and superior performance of the proposed approach for characterizing localized failure in quasi-brittle materials.

Keywords: Fracture; Finite element; Gradient damage; Strain localization; Quasi-brittle materials; Concrete

1. Introduction

1.1. Background

Quasi-brittle failure concerns a large number of geomaterials such as rock, concrete, or limestone, which are of great importance in many engineering applications, especially in infrastructure systems. Failure prediction for this class of quasi-brittle materials, which has fascinated researchers throughout the past decades, has been a major subject of study in the Computational Mechanics, and also a practical issue. Knowledge on the mechanical behavior of material degradation is indispensable, and tied to that objective, many relevant problems have been studied by scientists and researchers using all approaches including experimental, theoretical and computational methods, see for instance [1–4] and references therein.

The quasi-brittle materials inherently consist of microcracks and under severe loading conditions those microcracks may coalesce to macrocracks, resulting in a material softening phase, which significantly affects the performance of the media. Apart from experiment, the development of effective and accurate theoretical and computational models for prediction of material deterioration and rupture processes however remains a significant challenge in the scientific community. In the past decades, great efforts have been put into the loop of such development, and numerous progresses and results have been reported in the literature. Simulation and modeling of localized failure in quasi-brittle materials have been accomplished with different numerical methodologies. There are several approaches covering most existing works and they are grouped into two major categories: *smeared* damage approaches and *discrete* fracture mechanics approaches.

The discrete methods like the extended finite element method (XFEM) and cohesive crack model, e.g., [3,5,6], incorporate the discontinuities into the approximation of field variables, usually in conjunction with the linear elastic fracture mechanics (LEFM) theory, in which the concept of energy release rate based on the Griffith's theory is used. On the contrary, the smeared continuum damage approaches, e.g., [2,4,7], incorporate a damage variable into the constitutive model to represent the material softening and degradation, and also control its integrity. The discrete methods are suitable for tracking the macrocrack propagation, but have been proved to be less effective in multiple cracks problems, where initial cracks are challenging to be predicted. The regularized damage approaches, on the other hand, are able to model smeared fracture zones and do not require the initialization of cracks.

Apart from the popular regularized damage methods, the recently introduced phase field models, see for instance [8–10] and references therein, offer self-consistent descriptions of brittle fracture, which have been gaining popularity. The underlying idea of these phase field models is based on the energy minimization concept, wherein a damage variable or a phase field parameter is included to control the width of diffusive cracks or to let crack propagate along a path of least energy.

Being highly heterogeneous and anisotropic due to the complex compositions, quasi-brittle materials exhibit significantly different behaviors in tensile and compressive states. Hence, a proper continuum damage model needs its ability to capture these intricate material properties. For instance, anisotropic damage models, e.g., [2,11], describe the degradation process of materials by damage parameters driven by individual strain components. Another approach is the bi-dissipative damage model [4], which separately defines two damage parameters associated with the tensile and compressive loads. However, the implementation of these approach is often much more complicated and a large number of internal unknown variables are required. Whereas, in conventional isotropic damage models, e.g., see [12–15], the loss of material integrity is only represented by a single scalar variable, which is in turn derived from a scalar equivalent strain. Therefore, the isotropic damage models are much simpler and preferably developed for practical applications.