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Cyclic plasticity using Prager's translation rule and both nonlinear kinematic and isotropic hardening: Theory, validation and algorithmic implementation

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Abstract

Finite element analysis of structures under elasto-plastic nonproportional cyclic loadings is useful in seismic engineering, fatigue analysis and ductile fracture. Usual models with nonlinear stress–strain curves in cyclic behavior are based on Mróz multisurface plasticity, bounding surface models or models derived from the Armstrong–Frederick rule. These models depart from the associative Prager's rule with the main purpose of modeling aspects of cyclic nonlinear hardening. In this paper we develop a model for cyclic plasticity within the framework of the associative classical plasticity theory using Prager's rule accounting for anisotropic nonlinear kinematic hardening coupled with nonlinear isotropic hardening. We include the validation of the theory against several uniaxial and multiaxial cyclic experiments and an efficient fully implicit radial return algorithm. The parameters of the model are obtained directly by a discretization of the uniaxial stress–strain behavior. Remarkably, both the presented theory and the computational algorithm automatically recover classical bi-linear plasticity and the Krieg and Key algorithm if the user-prescribed stress–strain curve is bilinear.

Keywords: Cyclic plasticity; Associative plasticity; Radial return algorithm; Multisurface model; Nonlinear kinematic hardening

1. Introduction

The finite element analysis of structures under nonproportional and cyclic nonlinear behavior is very important in many applications, among them seismic engineering [1–3], fatigue and fracture analyses [4–7] and plastic forming [8–11]. The correct description of multiaxial hardening effects have proved to be critical for an accurate prediction of the effective displacements, accelerations and safety of the structures [12]. In fatigue analysis of notched specimens, the plastic loading at the notch edge induces nonproportional multiaxial loading [13], frequently studied through cyclic

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plasticity models which, thereafter, are related to nominal strains and stresses through incremental Neuber rules or related strain or energy methods [5,14–18].

In cyclic behavior, the accurate representation of the Bauschinger effect is crucial, a behavior which is approximately represented by Masing's rules. The classical linear kinematic hardening reproduces accurately such rules, has a very efficient integration algorithm and is available in most finite element programs. This model uses Prager's translation rule for the backstress, which is consistent with the principle of maximum dissipation [19]. However, a priori, the direct inclusion of a nonlinear anisotropic kinematic hardening function produces unphysical loops [20]. Therefore, finite element programs use Prager's rule exclusively in the case of *linear* kinematic hardening, and for nonlinear kinematic hardening, which is needed to better describe the cyclic loops, they resort to other types of formulations (if available to the user), typically based on the Armstrong and Frederick rule [21,22].

One of the procedures to account for anisotropic, history-dependent nonlinear kinematic hardening is based on the history of observable quantities by hereditary integrals, as in the endochronic theory, similar to those used in viscoelasticity [23]. However, since this approach is more cumbersome for computational application and for finite element analysis, the approach based on internal variables [20], where only information of the previous step is needed, is usually preferred.

Remarkably, the theories to model nonlinear kinematic hardening based on internal variables have departed from the classical linearly hardened framework, introducing different non-associative translation rules for the backstress, as Mróz's rule [24], Garud's rule [25] or the commented Armstrong and Frederick rule [26]. These rules have resulted in different models as the Mróz model [24], Chaboche's model [20,27], bounding surface models [1,28,29], and nonlinear kinematic hardening models with the addition of multiple backstresses [20,30]. The algorithmic implementation of some of these models is usually more elaborate than that of classical plasticity [31–35], and even though some recent efficient algorithms are available for some cases [21,22], they do not constitute a natural extension of classical Prager's plasticity. To improve the cyclic multiaxial plastic behavior, sometimes non-proportionality parameters, obtained for certain materials under certain loading paths by fitting experiments, are employed, like in Refs. [35–43]. However, despite the common belief [20, p. 206], it is possible to develop nonlinear kinematic hardening models using Prager's translation rule and preserving Masing's rules [44–47]. These models are similar in conception to Mroz's model, but with some crucial differences, as the preservation of Prager's rule, the use of a single yield surface (outer surfaces are hardening surfaces) and the simplicity of the integration algorithms. Furthermore, they lack the inconsistencies present in Mróz's model under multiaxial loading [48,49]. As we show in this paper, the predictions for multiaxial plastic behavior using Prager's rule are similar to the behavior observed in the experiments, at least for the different materials addressed below.

Despite the employed kinematic hardening rule, mixed kinematic–isotropic hardening models have been increasingly used in cyclic loading analysis [10,50–57]. For a better description of cyclic hardening/softening in certain materials, the effect of a memory for the strain amplitude can be included in the isotropic hardening rule [27,42,58,59], and the inclusion of isotropic cyclic softening may be used to model the effects of damage by fatigue. However, as seen below, the presence of isotropic hardening also modifies the anisotropic kinematic hardening moduli, an observation that must be considered in the formulation and computational algorithm.

Therefore, it is valuable, and the purpose of this paper, to extend the classical von Mises associative theory of plasticity using Prager's translation rule to account for history-dependent nonlinear anisotropic kinematic hardening combined with cyclic isotropic hardening/softening. In the next sections we first introduce the theory, showing that we just propose a simple extension of the classical framework by allowing a dependence of the effective hardening moduli on some internal variables. Then we introduce the method to compute such dependence, i.e. the effective hardening modulus at each instant. Thereafter we explain the efficient radial return algorithm, which for the bilinear case naturally recovers the solution from Krieg and Key [60]. With some examples we show that the theory predicts rather well the experimentally observed multiaxial behavior in several materials and that the finite element implementation preserves the asymptotic quadratic convergence of Newton schemes.

2. Classical J_2-plasticity using Prager's rule with nonlinear anisotropic mixed hardening

2.1. Classical associative plasticity with anisotropic, cyclically modified hardening

The stored "observable" (elastic) energy depends on the elastic strains $\boldsymbol{\varepsilon}_e$, which are a function of the total strains and some internal strains $\boldsymbol{\varepsilon}_p$ (the plastic strains), i.e. $\boldsymbol{\varepsilon}_e\left(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}_p\right)$. The chain rule derived from the explicit dependencies