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A geometrically regularized gradient-damage model with energetic equivalence

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Dedicated to Prof. Jie Li on the occasion of his 60th birthday

Highlights

- A geometrically regularized gradient-damage model with energetic equivalence is proposed.
- The damage evolution law emerges from the geometric regularization and energetic equivalence.
- The crack is implicitly represented, and cumbersome crack tracking strategies are not required.
- The length scale can be regarded either as a small numerical parameter or as a material property.
- The localization bandwidth approaches to a finite limit value, not exhibiting spurious damage growth.

Abstract

Aiming to bridge the gap between damage and fracture mechanics for modeling localized failure in solids, this paper addresses a novel *geometrically regularized* gradient-damage model with *energetic equivalence* for cracking evolution. With the free energy potential defined as usual in terms of the *local* strain and damage fields, the constitutive relations are derived consistently from the standard framework of thermodynamics. Upon the sharp crack topology *geometrically regularized* by the damage localization band with a length scale, the ensuing *energetic equivalence* naturally yields the damage evolution law of gradient-type and the associated boundary condition of Neumann-type. Compared to other gradient-damage models, no extra assumptions like the nonlocal energy residual and insulation condition are introduced. Moreover, the damage gradient, physically accounting for microscopic nonlocal interactions, is fully dissipative as expected. In line with the unified phase-field theory recently proposed by the author (Wu, 2017) during failure processes the material behavior is uniquely characterized by two constitutive functions, i.e., the degradation function defining the free energy potential of the bulk and the geometric function regularizing the sharp crack topology. In particular, optimal constitutive functions defining an equivalent cohesive zone model of general softening laws are postulated, with the involved parameters calibrated from standard material properties. The proposed model is numerically implemented into the multi-field finite element method and applied to several benchmark tests of concrete under mode-I and mixed-mode failure. It is found that the incorporated length scale can be regarded either a numerical parameter or a material property. For the former considered in this work, the length scale has negligible, if not no, effects on the global responses, so long as the sharp crack topology and the damage field of high gradients within the localization band are well resolved. Furthermore, the localization bandwidth does not exhibit spurious widening, but rather, it approaches to a finite value proportional to the length scale. Comparison between

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the numerical and experimental results, regarding the curve of load *versus* displacement and crack path, confirms validity of the proposed gradient-damage model for characterizing localized failure in quasi-brittle solids.

Keywords: Quasi-brittle solids; Localized failure; Gradient-damage model; Phase-field theory; Damage; Fracture

1. Introduction

Onset of macroscopic failure in solids and structures is generally preceded by highly localized non-uniform deformations (i.e., strain localization) within bands of small (or surfaces of negligible) width compared to the structural size. Inside and outside these localized domains, the strain field is either discontinuous due to continuous but non-smooth displacements or even singular (unbounded) caused by displacement discontinuities. Being such localized failure regarded as prognostics of catastrophic collapse of structures, it is of vital importance to predict its occurrence and quantify its adverse effects on overall structural responses.

Ever since the pioneering work of Ngo and Scordelis [1] and Rashid [2], a large volume of theoretical models based on fracture and damage mechanics have been proposed to characterize material behavior with softening regimes. Meanwhile, various computational approaches like the smeared and discrete crack methods have also been developed. However, just as commented by Prof. Bažant in the Speech of Acceptance of the 2009 Timoshenko Medal: "*The mechanics of damage and quasi-brittle fracture, with its scaling and interdisciplinary coupling, is a problem of the same dimension (as turbulence), which will not be closed even a century from now*", the modeling of localized failure in solids and structures is still an opening issue.

Among many others, the most stringent challenge arises when those theoretical models and computational methods are implemented into the numerical (discrete) context, e.g., the finite element method (FEM) or other alike. On the one hand, owing to the lack of a well-defined length scale and the loss of ellipticity in the governing equations, local material models with softening regimes cannot be used straightforwardly in the numerical context; otherwise, mesh dependent and misleading results would be unavoidable [3,4]. In order to circumvent such issues, non-local and gradient-enhanced damage models [5,6] have been proposed. With a nonlocal damage driving force defined either in integral or gradient form, a material length scale is introduced to restore well-posedness of the resulting (initial) boundary value problem. However, the boundary effect related to the integral form [7] and the boundary condition for the gradient form [8] cannot be easily dealt with. Even worse, as a constant interaction domain allows the energy transferring from the fracture process zone to neighboring regions in elastic unloading, non-local and gradient-enhanced damage models tend to diffuse the localization band [9], leading to spurious damage growth. Consequently, localized deformations are precluded due to this 'damage diffusion' process, giving erroneous numerical predictions in mode-I failure and shear band problems [10]. Interesting remedies include the 'over-nonlocal' formulation [11,12] and nonlocal models with stress-based weighting functions [13], decreasing interactions [14] or evolving length scale [15], etc.; see [16] and the references therein.

On the other hand, due to the intrinsic kinematic deficiency and *a priori* limitation of crack propagation paths, respectively, the classical smeared and discrete crack methods suffer from the issues of mesh-alignment dependence and spurious stress locking [3,17]. In order to restore objectivity of the numerical solution, the more powerful enriched FEM, e.g., the embedded FEM [18] and extended FEM [19], etc., were proposed; see [20,21] for the reviews. With the discontinuity kinematics enhanced and the solution space enriched, such advanced numerical methods allow cracks propagating arbitrarily intra-elements and stress locking-free states can be achieved for a fully softened crack, provided the crack path is well tracked during the entire failure process. However, the numerical issues related to the ill-conditioned system matrix [22], oscillations in tractions [23], quadrature of discontinuous functions [24], etc., limit the application of such methods, though some remedies like the stable XFEM [25,26], the regularized XFEM [27], etc., were developed. Even worse, it is rather tedious, if not impossible, to explicitly represent the crack geometry and track the propagation path, particularly for non-smooth crack surfaces with branching and merging in 3-D cases [28].

Regarding the above facts, some regularized approaches have been advocated during the last two decades, trying to reconcile damage and fracture mechanics, or computationally, the smeared and discrete crack methods. Both being thermodynamically consistent, the thick level set (TLS) damage model and the phase-field model (PFM),