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S. Bertoluzza, V. Chabannes, C. Prud'homme, M. Szopos

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BOUNDARY CONDITIONS INVOLVING PRESSURE FOR THE STOKES PROBLEM AND APPLICATIONS IN COMPUTATIONAL HEMODYNAMICS

S. BERTOLUZZA

CNR - Istituto di Matematica Applicata e Tecnologie Informatiche "Enrico Magenes", v. Ferrata 1, 27100 Pavia (Italy)

V. CHABANNES, C. PRUD'HOMME, M. SZOPOS

Université de Strasbourg, CNRS, IRMA UMR 7501, F-67000 Strasbourg, France

Abstract. Pressure driven flows typically occur in hydraulic networks, *e.g.* oil ducts, water supply, biological flows, microfluidic channels *etc.* However, Stokes and Navier-Stokes problems are most often studied in a framework where Dirichlet type boundary conditions on the velocity field are imposed, thanks to the simpler settings from the theoretical and numerical points of view. In this work, we propose a novel formulation of the Stokes system with pressure boundary condition, together with no tangential flow, on a part of the boundary in a standard Stokes functional framework using Lagrange multipliers to enforce the latter constraint on velocity. More precisely, we carry out *(i)* a complete analysis of the formulation from the continuous to discrete level in two and three dimensions *(ii)* the description of our solution strategy, *(iii)* a verification of the convergence properties with an analytic solution and finally *(iv)* three-dimensional simulations of blood flow in the cerebral venous network that are in line with *in-vivo* measurements and the presentation of some performance metrics with respect to our solution strategy.

1. Introduction

Stokes and Navier-Stokes problems are often studied in a framework where Dirichlet type boundary conditions on the velocity field are imposed. However, in hydraulic network-like systems, for instance oil ducts, water supply, microfluidic channels or the blood circulatory system, different formulations with boundary conditions involving components of the velocity field, stresses or pressure, are of interest. A recent review of some of the formulations proposed in the literature, and their associated boundary conditions, with a focus on applications to air and blood flows can be found in [17].

In this work, we are motivated by the computational modeling of some biological flows driven by physiological pressure gradients and more precisely we are interested in the case when the velocity field is imposed on one part of the boundary and pressure values are prescribed, together with the condition of no tangential flow, on the remaining part. A variational formulation taking into account this type of boundary conditions was first introduced in the seminal works [29, 6, 11]. A lot of subsequent literature was devoted to this topic. At a continuous level, the well-posedness of the variational formulation both for Stokes and Navier-Stokes systems was investigated in a steady Hilbertian case [11, 20], unsteady nonlinear two-dimensional case [26] and recently extended to L^p -theory for $1 < p < \infty$ in [2]. Several discretization approaches were proposed, including finite differences [24], SPH method [23] or finite elements [12, 20, 5, 7]. Numerical experiments in the finite element framework enforce this type of boundary conditions through a penalty method, for Newtonian [12, 5] and generalized Newtonian fluids [5]. Recent developments concern the Navier-Stokes problem in the context of a simplified fluid-structure model for blood flows [21]. This so-called *Surface Pressure Model* is analyzed in [10].

Key words and phrases. Stokes equations, pressure boundary conditions, finite element method, computational hemodynamics.