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Sebastián Acosta

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High order surface radiation conditions for time–harmonic waves in exterior domains

Sebastián Acosta[a]

[a]*Department of Pediatrics, Baylor College of Medicine and Texas Children's Hospital, TX*

Abstract

We formulate a new family of high order on–surface radiation conditions to approximate the outgoing solution to the Helmholtz equation in exterior domains. Motivated by the pseudo–differential expansion of the Dirichlet–to–Neumann operator developed by Antoine et al. (J. Math. Anal. Appl. 229:184–211, 1999), we design a systematic procedure to apply pseudo–differential symbols of arbitrarily high order. Numerical results are presented to illustrate the performance of the proposed method for solving both the Dirichlet and the Neumann boundary value problems. Possible improvements and extensions are also discussed.

Keywords: On–surface radiation conditions, semi–analytical approximations, absorbing boundary conditions, wave scattering, Helmholtz equation
2010 MSC: 35J05, 58J40, 58J05, 41A60, 65N38

1. Introduction

This paper is concerned with the approximation of the outgoing solution to the Helmholtz equation in a domain Ω exterior to a closed surface Γ. We consider both the Dirichlet and the Neumann boundary value problems,

$$\Delta u + k^2 u = 0 \qquad \text{in } \Omega, \tag{1a}$$

$$u = f_{\text{Dir}} \quad \text{or} \quad \partial_\nu u = f_{\text{Neu}} \qquad \text{on } \Gamma, \tag{1b}$$

$$\lim_{r\to\infty} r\left(\partial_r u - iku\right) = 0, \tag{1c}$$

where $r = |\mathbf{x}|$ and $\mathbf{x} \in \mathbb{R}^3$. The main underlying assumptions are that $k > 0$ is constant in Ω, that Γ is sufficiently smooth, and that the domain enclosed by Γ is strictly convex. These assumptions imply that the wave field in Ω is purely outgoing, ruling out the possibility of waves reflecting back and forth between disjoint subsets of Γ.

When exact analytical solutions to the boundary value problem (1) are not available, boundary integral equations and their discretizations offer some of the most robust choices to approximate the solution numerically [1, 2, 3, 4]. Another approach is to truncate and discretize the domain surrounding Γ and impose an absorbing condition on an artificial boundary to approximate the

Email address: sebastian.acosta@bcm.edu (Sebastián Acosta)
URL: https://sites.google.com/site/acostasebastian01 (Sebastián Acosta)