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Daniel Elfverson, Mats G. Larson, Axel Målqvist

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Multiscale methods for problems with complex geometry

Daniel Elfverson*, Mats G. Larson*, and Axel Målqvist†

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Abstract

We propose a multiscale method for elliptic problems on complex domains, e.g. domains with cracks or complicated boundary. For local singularities this paper also offers a discrete alternative to enrichment techniques such as XFEM. We construct corrected coarse test and trail spaces which takes the fine scale features of the computational domain into account. The corrections only need to be computed in regions surrounding fine scale geometric features. We achieve linear convergence rate in the energy norm for the multiscale solution. Moreover, the conditioning of the resulting matrices is not affected by the way the domain boundary cuts the coarse elements in the background mesh. The analytical findings are verified in a series of numerical experiments.

1 Introduction

Partial differential equations with data varying on multiple scales in space and time, so called *multiscale problems*, appear in many areas of science and engineering. Two of the most prominent examples are composite materials and flow in a porous medium. Standard numerical techniques may perform arbitrarily bad for multiscale problems, since the convergence rely on smoothness of the solution [6]. Also adaptive techniques [29], where local singularities are resolved by local mesh refinement, fail for multiscale problems since the roughness of the data is often not localized in space. As a remedy against this issue generalized finite element methods and other related multiscale techniques have been developed [3, 18, 19, 17, 9, 19, 22, 23, 25, 5, 4]. So far these techniques have focused on multiscale coefficients in general and multiscale diffusion in particular. Significantly less work has been directed towards handling a computational domain with multiscale boundary, see e.g. [1]. However, in many applications including voids and cracks in materials and rough surfaces, multiscale behavior emanates from the complex geometry of the computational domain. Furthermore, the classical multiscale methods mentioned above aim at, in different ways, upscaling the multiscale data to a coarse scale where it is possible to solve the equation to a reasonable computational cost. However, these techniques typically assume that the representation of the computational domain is the same on the coarse and fine scale. In practice this is very difficult to achieve unless the computational domain has a simple shape, which is not the case in many practical applications.

In this paper we design a multiscale method for problems with complex computational domain. In order to simplify the presentation we neglect multiscale coefficients in the analysis even though the methodology directly extends to this situation. The proposed algorithm is based on the localized orthogonal decomposition (LOD) technique presented in [23] and further developed in [10, 11, 24, 27]. In LOD both test and trail spaces are decomposed into a multiscale space and a remainder space that are orthogonal with respect to the scalar product induced by the

*Department of Mathematics, Umeå University, SE-901 87 Umeå, Sweden. Supported by SSF.
†Department of Mathematical Sciences, Chalmers University of Technology and University of Gothenburg SE-412 96 Gööteborg, Sweden. Supported by the Swedish research council and SSF.