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Non-uniform rational Lagrange functions and its applications to isogeometric analysis of in-plane and flexural vibration of thin plates

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Abstract

New basis functions were developed for isogeometric analysis (IGA) to overcome the difficulties of IGA using NURBS (Non-Uniform Rational B-Splines) on coping with Dirichlet boundary conditions. The new basis functions were constructed through nesting rational local Lagrange interpolations like the T-spline and were evaluated in similar procedure as the finite difference method. Explicit expressions for the new basis functions were presented. Due to their equivalence to the NURBS, the new basis functions were named as non-uniform rational Lagrange (NURL) basis. IGA using NURL includes the finite element method as a special case but the geometry in IGA using NURL is exact. IGA using NURL can carry out p-refinement that has the nesting feature of the k-refinement of NURBS. Dirichlet boundary conditions can be directly imposed in IGA using NURL because the NURL are interpolation basis functions. A method of directly transforming the tensor product basis of triangular patches to area coordinates was presented and the singularity problem at the edge degenerated to a single point was solved. The methods developed in this work were applied to in-plane and flexural vibration of thin plates. Comparisons with available results in literatures showed the fast convergence and high accuracy of IGA using NURL and the transformation method for triangular patches. Introduction of a MATLAB toolbox of the NURL was appended.

Keywords: Isogeometric analysis; Finite element method; Local interpolation; Triangular patches; In-plane vibration; Flexural vibration

1. Introduction

In this work we present alternative basis that are interpolation functions and can equivalently represent NURBS (Non-Uniform Rational B-Splines) geometries for isogeometric analysis (IGA). The basis presented in this work

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had many features in common with the finite difference method and were constructed through nesting rational local Lagrange interpolations like the T-spline [1,2]. The new basis functions were named as non-uniform rational Lagrange (NURL) basis due to their equivalence to the NURBS. Isogeometric analysis using NURL basis faithfully adheres to the isoparametric concept and geometric exactness of IGA, and includes the finite element method (FEM) as a special case. Therefore, the limitation of IGA using NURBS that has some features in common with the mesh-free methods can be overcome. This work also presented a method of coping with triangular patches for IGA and discussed the potential of the NURL basis in coping with C^1 continuity between patches.

The concept of isogeometric analysis (IGA) was proposed by Hughes [3,4] and was attempted to directly use the CAD geometry and entirely eliminate the finite element polynomial description. A coarse mesh of "non-uniform rational B-splines (NURBS) elements" was used to match the exact CAD geometry. This bridges the gap between geometry and analysis. The solution space for dependent variables is represented in terms of the same functions that represent the geometry. For purposes of analysis, the basis is refined and/or its degree is elevated without changing the geometry or its parameterization. This process does not require any further communication with the CAD system and therefore IGA was expected to facilitate more widespread adoption in industry. Since it was proposed about 10 years ago [3], IGA has been widely used in many fields of engineering and science, and enormous papers using or developing IGA have been published, for example, [5–8]. However, the NURBS are non-interpolation basis and the imposition of Dirichlet boundary conditions is not as easy as with an interpolatory basis, although there are plenty of good (local) quasi-interpolation schemes which are perfectly suited for imposing boundary conditions [9–12].

The equivalence of NURBS with other types of piecewise polynomials has been discussed in [13]. However, the advantages of NURBS in computer aided design (CAD) were addressed and the potential of other types of polynomials for analysis was not considered. Since finite element method typically uses Lagrange polynomials, it is necessary to directly transform NURBS geometries into NURL geometries. Then isogeometric analysis can be carried out in the common framework of FEM. Since the transformation is devoted for analysis instead of geometric modification, it is necessary and beneficial. The B-splines are piecewise polynomials defined on knot vectors with multiplicity. The knot vectors contain the intervals of the piecewise polynomials and the continuous property of the geometry. Due to the uniqueness of polynomial interpolation in each interval, a pth degree B-spline curve can be exactly represented by Lagrange polynomials using $p + 1$ nodes on each interval of the curve. Thus, Lagrange polynomials in rational form can exactly represent NURBS curves in each interval. Similar to NURBS, NURL surfaces and volumes can be obtained by tensor product of one dimensional NURL basis. The Lagrange extraction and projection for NURBS basis functions by Schillinger et al. [14] presented a direct link between isogeometric and standard nodal finite element formulations. This leads to a C^0 basis that is able to represent NURBS. Instead of increasing C^0 segment as [14] during refinements, one can increase the number of nodes in an interval and approximate an arbitrary point in the interval through $p + 1$ nodes close to the point in the interval. This procedure is similar to the finite difference method (FDM) or the T-spline [1,2]. This leads to the NURL basis functions of this work formed by nesting local Lagrange interpolation functions. We can see that the NURL basis functions differ from the Lagrange extraction and projection for NURBS basis functions by Schillinger et al. [14] only by a searching of evaluation domain. This simple change does not increase the computational cost and complexity while makes the NURL spaces nested, and therefore makes k-refinement be possible for the NURL, which has been shown to more advantageous than the common h- and p-refinement [3,4]. In similar way as the NURL, non-uniform rational Hermite (NURH) basis can also be constructed through nesting rational local Hermite interpolation.

Through the discussion above, the value of the NURL compared with the NURBS or the standard (rational) Lagrange interpolation basis can be summarized as follows:

(1) The NURL is local basis. So it can avoid the problems caused by global Lagrange interpolation of high order p-version finite element method. Due to its nested definition that permits k-refinement, the NURL can provide results with much better accuracy compared with the h-version finite element method. The convergence and accuracy of the NURL are similar to the NURBS as can be seen from the provided numerical examples in this work.
(2) Compared with the NURBS, the NURL is interpolation basis, so the imposition of Dirichlet boundary conditions is straightforward for the NURL. Due to its equivalence with the NURBS and its simplicity of usage, the NURL provides an alternative method of geometric computation.