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# Adaptive Finite Element Method for Fractional Differential Equations using Hierarchical Matrices☆

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**Abstract**

A robust and fast solver for the fractional differential equation (FDEs) involving the Riesz fractional derivative is developed using an adaptive finite element method. It is based on the utilization of hierarchical matrices ($\mathcal{H}$-Matrices) for the representation of the stiffness matrix resulting from the finite element discretization of the FDEs. We employ a geometric multigrid method for the solution of the algebraic system of equations. We combine it with an adaptive algorithm based on a posteriori error estimation A posteriori error estimation based adaptive algorithm is used to deal with general-type singularities arising in the solution of the FDEs. Through various test examples we demonstrate the efficiency of the method and the high-accuracy of the numerical solution even in the presence of singularities. The proposed technique has been verified effectively through fundamental examples including Riesz, Left/Right Riemann-Liouville fractional derivative and, furthermore, it can be readily extended to more general fractional differential equations with different boundary conditions

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