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Fatma G. Eroglu, Songul Kaya, Leo G. Rebholz

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A Modular Regularized Variational Multiscale Proper Orthogonal Decomposition for Incompressible Flows

Fatma G. Eroglu [*]   Songul Kaya [†]   Leo G. Rebholz [‡]

Abstract

In this paper, we propose, analyze and test a post-processing implementation of a projection-based variational multiscale (VMS) method with proper orthogonal decomposition (POD) for the incompressible Navier-Stokes equations. The projection-based VMS stabilization is added as a separate post-processing step to the standard POD approximation, and since the stabilization step is completely decoupled, the method can easily be incorporated into existing codes, and stabilization parameters can be tuned independent from the time evolution step. We present a theoretical analysis of the method, and give results for several numerical tests on benchmark problems which both illustrate the theory and show the proposed method's effectiveness.

Keywords: proper orthogonal decomposition, projection-based variational multiscale, reduced order models, post-processing

1 Introduction

We consider the incompressible Navier-Stokes equations (NSE) on a polyhedral domain $\Omega \subset \mathbb{R}^d, d \in \{2, 3\}$ with boundary $\partial\Omega$:

$$\begin{aligned} \mathbf{u}_t - \nu\Delta\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p &= \mathbf{f} && \text{in } (0,T]\times\Omega, \\ \nabla\cdot\mathbf{u} &= 0 && \text{in } [0,T]\times\Omega, \\ \mathbf{u} &= \mathbf{0} && \text{in } [0,T]\times\partial\Omega, \\ \mathbf{u}(0,\mathbf{x}) &= \mathbf{u}_0 && \text{in } \Omega, \\ \int_\Omega p\, d\mathbf{x} &= 0, && \text{in } (0,T]. \end{aligned} \tag{1.1}$$

Here, $\mathbf{u}(t,\mathbf{x})$ is the fluid velocity and $p(t,\mathbf{x})$ the fluid pressure. The parameters in (1.1) are the kinematic viscosity $\nu > 0$, inversely proportional to the Reynolds number, $Re = \mathcal{O}(\nu^{-1})$, the prescribed body forces $\mathbf{f}(t,\mathbf{x})$ and the initial velocity field $\mathbf{u}_0(\mathbf{x})$. It is known that due to the wide range of scales in many complex fluid flows, simulating these flows by a direct numerical simulation (DNS) can be very expensive, and sometimes is even infeasible - from the Kolmogorov 1941 theory, it is known that a resolved DNS requires $O(Re^{9/4})$ meshpoints [3]. To make the situation even more

[*]Department of Mathematics, Middle East Technical University, 06800 Ankara, Turkey; fguler@metu.edu.tr
[†]Department of Mathematics, Middle East Technical University, 06800 Ankara, Turkey; smerdan@metu.edu.tr
[‡]Department of Mathematical Sciences, Clemson University, Clemson, SC, 29634; rebholz@clemson.edu; Partially supported by NSF DMS1522191 and U.S. Army Grant 65294-MA.