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Computational Two-mode Asymptotic Bifurcation Theory Combined with Hyper Dual Numbers and Applied to Plate/Shell Buckling

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Abstract

For error-free computation of higher-order derivatives of a complex mathematical expression composed of elementary functions, hyper-dual numbers (HDNs) are receiving close attention in computational engineering and sciences. Differently from conventional finite differences, HDNs numerically evaluate, for instance, the exact first-order and second-order derivatives of the stiffness matrix with respect to nodal degrees-of-freedom (dof) in nonlinear finite element (FE) analysis.

One of the most promising applications of HDNs is the asymptotic structural stability theory, which usually requires, in conventional series expansions, an unrealistically large number of second-order derivative of the stiffness matrix to predict the instability behavior of large-scale FE models. This computational bottleneck may be avoided along with so-called classical Lyapunov-Schmidt-Koiter condensation by a novel idea proposed in the present study. The proposed two-mode computational asymptotic theory is implementable in engineering practice to algebraically identify snap-through and path-branching in stability problems. One more associated innovation is a graphical solution of two simultaneous equations resulting from the asymptotic expansions. This sophisticated solution idea utilizing a popular mathematical tool (MATLAB) is widely applicable to quickly visualize the location and number of existing solutions.

More specifically, the present paper formulates a two-mode asymptotic bifurcation theory in combination with HDNs and presents an examination of the proposed computational theory on stability problems of plates and shells. Near a precisely computed singular point on the equilibrium path, the incremental displacement field is represented in two modes: $\boldsymbol{\theta}$ and $\boldsymbol{\delta}$. One mode ($\boldsymbol{\theta}$) is the critical eigenvector corresponding to the zero eigenvalue of the singular tangent stiffness matrix. The other one ($\boldsymbol{\delta}$) is a linear combination of all other non-critical eigenvectors. $\boldsymbol{\theta}$ and $\boldsymbol{\delta}$ respectively constitute the homogeneous and particular solutions of the linearized equilibrium equations established in the singular point. Two perturbation parameters, σ and τ, work as weights of $\boldsymbol{\theta}$ and $\boldsymbol{\delta}$, respectively, to find possible equilibria around the singular point. The nonlinear equilibrium equations are expanded into asymptotic series of σ and τ to use HDNs to compute 20 coefficients including first-order and second-order derivatives of the stiffness matrix in polynomials. The simultaneous polynomial equations of σ and τ are solved by using the MATLAB graphic operations.

The obtained results in numerical examples well predict the stability behavior in excellent agreement with solutions available in conventional FE stability analysis. The two-mode asymptotic bifurcation theory combined with HDNs is sufficient to diagnose snap-through, asymmetric branching, unstable and stable symmetric bifurcation in structural stability problems.

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