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Multiscale Hybrid-Mixed Method for the Stokes and Brinkman Equations – The Method

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Abstract

The multiscale hybrid-mixed (MHM) method is extended to the Stokes and Brinkman equations with highly heterogeneous coefficients. The approach is constructive. We first propose an equivalent dual-hybrid formulation of the original problem using a coarse partition of the heterogeneous domain. Faces may be not aligned with jumps in the data. Then, the exact velocity and the pressure are characterized as the solution of a global face problem and the solutions of local independent Stokes (or Brinkman) problems at the continuous level. Owing to this decomposition, the one-level MHM method stems from the standard Galerkin approach for the Lagrange multiplier space. Basis functions are responsible for upscaling the unresolved scales of the medium into the global formulation. They are the exact solution of the local problems with prescribed Neumann boundary conditions on faces driven by the Lagrange multipliers. We make the MHM method effective by adopting the unusual stabilized finite element method to solve the local problems approximately. As such, equal-order interpolation turns out to be an option for the velocity, the pressure and the Lagrange multipliers. The numerical solutions share the important properties of the continuum, such as local equilibrium with respect to external forces and local mass conservation. Several academic and highly heterogeneous tests infer that the method achieves super-convergence for the velocity as well optimal convergence for the pressure and also for the stress tensor in their natural norms.

Keywords: Stokes equation, Brinkman model, mixed method, hybrid method, multiscale finite element
2010 MSC: 65N12, 65N30

1. Introduction

Interesting problems are modeled through the Stokes (Brinkman) operator with coefficients that account for highly heterogenous media. Diverse engineering, geophysics, and hydrology problems involve such a system in which the common characteristic is the wide range of length-scales presented. To be more precise, let $\Omega \subset \mathbb{R}^d$, $d \in \{2, 3\}$, be an open, bounded domain with polygonal boundary $\partial\Omega$. The generalized Stokes problem, also called the Brinkman model, is: find the velocity $\boldsymbol{u}$ and the pressure p such that

$$\begin{aligned} -\nu\,\Delta \boldsymbol{u} + \boldsymbol{\gamma}\,\boldsymbol{u} + \nabla p &= \boldsymbol{f} \quad \text{in } \Omega, \\ \nabla \cdot \boldsymbol{u} &= 0 \quad \text{in } \Omega, \\ \boldsymbol{u} &= \boldsymbol{g} \quad \text{on } \partial\Omega, \end{aligned} \tag{1}$$

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