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# Isogeometric shape optimization of smoothed petal auxetic structures via computational periodic homogenization

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**Abstract**

An important feature that drives the auxetic behaviour of the star-shaped auxetic structures is the hinge-functional connection at the vertex connections. This feature poses a great challenge for manufacturing and may lead to significant stress concentrations. To overcome these problems, we introduced smoothed petal-shaped auxetic structures, where the hinges are replaced by smoothed connections. To accommodate the curved features of the petal-shaped auxetics, a parametrisation modelling scheme using multiple NURBS patches is proposed. Next, an integrated shape design frame work using isogeometric analysis is adopted to improve the structural performance. To ensure a minimum thickness for each member, a geometry sizing constraint is imposed via piece-wise bounding polynomials. This geometry sizing constraint, in the context of isogeometric shape optimization, is particularly interesting due to the non-interpolatory nature of NURBS basis. The effective Poisson ratio is used directly as the objective function, and an adjoint sensitivity analysis is carried out. The optimized designs – smoothed petal auxetic structures – are shown to achieve low negative Poisson's ratios, while the difficulties of manufacturing the hinges are avoided. For the case with six petals, an in-plane isotropy is achieved.


*Keywords:* Isogeometric shape optimization; Auxetic material; Negative Poisson's ratio; Sizing constraint; Computational periodic homogenization; Adjoint method

## 1. Introduction

In contrast to conventional materials that contract (expand) transversely when stretched (compressed), auxetic materials expand (contract) perpendicularly to the stretching (compression) direction, leading to a so-called negative Poisson's ratio (NPR). This auxetic behaviour leads to interesting performance in energy absorption, shear, indentation, damping, acoustics and crushing, with many potential applications in the civil and military sectors [1–7].

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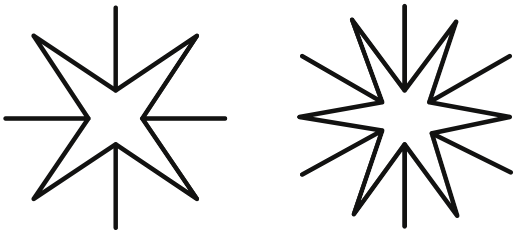

**Fig. 1.** Star-shaped structures with 4 and 6 vertices [40,41,6].

The design of auxetic (micro-) structures has attracted research interests for more than three decades. In 1985, Almgren presented a structure with Poisson's ratio of −1 using rods, hinges, and springs [8]. Conceptual designs of composite materials with Poisson's ratio close to −1 were presented in [9]. Some important features of auxetic materials, such as the re-entrant corners, were discussed in the works of Lakes [10], Friis et al. [11], Evans [12], and Phan-Thien and Karihaloo [13]. A separate class of 3D auxetics that exploits the buckling mechanism in structures is proposed by [14]. Design techniques using modern numerical methods such as topology optimization started with the work of Sigmund [15], in which a topology optimization framework for designing 2D and 3D auxetic truss-based structures was presented. The work was next extended into a 2D continuum-based approach in [16] and further developed into a manufacturable compliant micro-mechanism with negative Poisson's ratio in [17].

Since these pioneering work, different techniques have been adopted for auxetic structure design optimization. Radman [18] studied the topology design for auxetic structures using bi-directional evolutionary structural optimization (BESO) method [19–21]. Wang and co-workers [22,23] presented some design optimization works using level-set method [24–26]. Schwerdtfeger et al. [27] utilized topology optimization to improve a given 3D auxetic structure, such that the resulting design is acceptable for the manufacturing process. Kureta and Kanno [28] studied the auxetic structure design using a mixed integer programming approach. Kaminakis and Stavroulakis [29] presented an evolutionary-hybrid algorithm to perform the design optimization. Using genetic algorithm and differential evolutionary algorithm, a unit cell of disordered rib grids-based structures is optimized to achieve a negative Poisson ratio in the works of Javadi et al. [30] and Horrigan et al. [31]. Isotropic design optimization for auxetic material is studied recently by Czarnecki and Wawruch [32]. Recent efforts incorporating geometrical non-linearity and manufacturability constraint in the design optimization can be found in the works of Wang et al. [33] and Clausen et al. [34], with the resulting structures having high geometrical simplicity, design flexibility, and manufacturability. Other design techniques for molecular structures and multi-scale problems can be found in [35–39].

In this paper, we focus on the class of star-shaped auxetic structures depicted in Fig. 1, which has been studied by several researchers [e.g. [40–42]]. Generally, the auxetic performance of a star-shaped structure is dependent on the flexibility at the vertices. Hinge-like connections, while beneficial, cannot be manufactured easily. Continuous vertices are amenable to the manufacturing process, though the sharp corners induce a stress concentration effect. Moreover, the rigidity of connections at vertices degrade the auxetic performance of a star-shaped structure, to be discussed later in Section 2.

Following this concept, we propose a series of smoothed petal conceptual designs, examples as shown in Fig. 2, based on the smoothening of connections in the reference star-shaped structures. With an appropriate shape design technique, a reasonably good auxetic behaviour of these conceptual designs can be achieved, without the manufacturing difficulties associated with hinge-like connections.

A key challenge for the shape design of the proposed smoothed petal structures is the proper integration between the curved geometrical description and the corresponding finite element analysis. Using non-uniform rational b-spline (NURBS) basis function as the shape function, the isogeometric analysis (IGA) [43] is well suited for shape optimization in terms of the exact geometrical description and enhanced sensitivity analysis [44,45]. Shape optimization based IGA has been adopted for curved beam structures [46,47], vibrating membranes [48], fluid mechanics [49], shells [50,51], photonic crystals [52], Stokes flow problems [53], etc. Other works using NURBS as a tool for shape optimization can be found in [54–57].

Shape optimization has been utilized for the design of auxetics [e.g. [33,34]]. Departing from these established approaches, this paper adopts isogeometric analysis for shape design optimization of auxetic structures such that