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A conservative nonlocal convection-diffusion model and asymptotically compatible finite difference discretization

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Abstract

In this paper, we first propose a nonlocal convection-diffusion model, in which the convection term is constructed in a special upwind manner so that mass conservation and maximum principle are maintained in any space dimension. The well-posedness of the proposed nonlocal model and its convergence to the classical local convection-diffusion model are established. A quadrature-based finite difference discretization is then developed to numerically solve the nonlocal problem and it is shown to be consistent and unconditionally stable. We further demonstrate that the numerical scheme is asymptotically compatible, that is, the approximate solutions converge to the exact solution of the corresponding local problem when $\delta \to 0$ and $h \to 0$. Numerical experiments are also performed to complement the theoretical analysis.

Keywords: Nonlocal convection-diffusion, mass conservation, maximum principle, quadrature-based finite difference, asymptotically compatible

1. Introduction

There have been much recent interests in nonlocal models given in the form of partial integral equations. Existing studies range from applications in nonlocal mechanics [2, 23, 26, 34, 38, 37, 39, 46], diffusion [24, 30, 33, 40], image analysis [1, 20, 21], biology [4, 31] to mathematical analysis and computational simulations [3, 5, 8, 9, 11, 12, 14, 18, 22, 45, 27, 41, 48]. Our goal here is to propose a new conservative nonlocal convection-diffusion model and discuss its effective numerical solution. While past studies on models of nonlocal diffusion have largely focused on pure symmetric processes where nonlocal interactions have no orientation bias towards any particular direction, the number of studies on nonlocal convection-diffusion models is also growing. In [13] a nonlinear nonlocal convection model is presented as a one-dimensional nonlocal balance law. A modified version was given in [10] based on the idea of monotone schemes. Several studies of fractional convection-diffusion models may also be viewed as works on nonlocal models, see [19, 28] and the references

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