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A nonlinear consistent penalty method weakly enforcing positivity in the finite element approximation of the transport equation

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Abstract

We devise and analyze a new stabilized finite element method to solve the first-order transport (or advection-reaction) equation. The method combines the usual Galerkin/Least-Squares approach to achieve stability with a nonlinear consistent penalty term inspired by recent discretizations of contact problems to weakly enforce a positivity condition on the discrete solution. We prove the existence and the uniqueness of the discrete solution. Then we establish quasi-optimal error estimates for smooth solutions bounding the usual error terms in the Galerkin/Least-Squares error analysis together with the violation of the maximum principle by the discrete solution. Numerical examples are presented to illustrate the performances of the method.

Key words: stabilized finite element method, consistent penalty, positivity preserving, transport equation, discrete maximum principle

1 Introduction

The design of robust and accurate finite element methods for first-order transport (or advection-reaction) equations or for advection-dominated advection-diffusion equations remains an active field of research. Indeed, the task of designing a numerical scheme that is of higher order than one in the zone where the exact solution is smooth, but preserves the monotonicity properties of the exact solution on the discrete level, is nontrivial. Since it is known that such a scheme necessarily must be nonlinear even for linear equations, one typical strategy adopted when working with stabilized finite element methods is to add an additional nonlinear shock-capturing term, designed to make the