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Computationally efficient beam elements for accurate stresses in sandwich laminates and laminated composites with delaminations

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Abstract

Laminated composites are prone to delamination failure due to the lack of reinforcement through the thickness. Therefore, during the design process the initiation and propagation of delaminations should be accounted for as early as possible. This paper presents computationally efficient nine degree-of-freedom (dof) and eight-dof shear locking-free beam elements using the mixed form of the refined zigzag theory ($\text{RZT}^{(m)}$). The corresponding nine-dof and eight-dof elements use the anisoparametric and constrained anisoparametric interpolation schemes, respectively, to eliminate shear locking in slender beams. The advantage of the present element over previous RZT beam elements is that no post-processing is required to accurately model the transverse shear stress while maintaining the computational efficiency of a low-order beam element. Comparisons with high-fidelity finite element models and three-dimensional elasticity solutions show that the elements can robustly and accurately predict the displacement field, axial stress and transverse shear stress through the thickness of a sandwich beam or a composite laminate with an embedded delamination. In fact, the accuracy and computational efficiency of predicting stresses in laminates with embedded delaminations make the present elements attractive choices for RZT-based delamination initiation and propagation methodologies available in the literature.

Keywords: Shear locking; Zigzag theory; Reissner's mixed variational theorem; Delamination

1. Introduction

Laminated composites are prone to delamination failure due to the lack of reinforcement through the thickness, and this failure mode adversely affects the structural integrity of composite structures. Hence, the initiation and propagation of delaminations should be accounted for at the early stages in the design process. In this respect, tools for accurate stress predictions are an important prerequisite.

Currently, the standard approach in industry is to use three-dimensional finite element (3-D FE) models or layerwise theories to predict accurate 3-D stress fields. At the preliminary design stage, detailed yet computationally expensive

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3-D FE solutions are prohibitive for rapid design as meshes with multiple elements per layer are typically required for converged results. Therefore, 3-D layerwise models are often only used on a component-scale level in areas of high stress concentration or for safety-critical components.

For most composite laminates, the thickness dimension is at least an order of magnitude smaller than representative in-plane dimensions, which allows these structures to be modeled as thin beams, plates or shells. This feature facilitates a reduction from a 3-D problem to a 2-D one coincident with a chosen reference axis or surface. The major advantage of this approximation is a significant reduction in the total number of variables and computational effort required.

In multi-layered composite structures, the effects of transverse shear and normal deformations are especially pronounced because the ratios of longitudinal to transverse moduli are approximately one order of magnitude greater than for isotropic materials ($E_{xx}^{iso}/G_{xz}^{iso} = 2.6,\ E_{11}/G_{13} \approx 140/5 = 28$ and $E_{xx}^{iso}/E_{zz}^{iso} = 1,\ E_{11}/E_{33} \approx 150/10 = 15$). Second, differences in layerwise transverse shear and normal moduli lead to abrupt changes in the slopes of the three displacement fields u_x, u_y, u_z at layer interfaces. This is known as the zigzag phenomenon (see Fig. 1) and, as shown by Demasi [1], the zigzag form of the displacements u_x, u_y and u_z can be derived directly from interfacial continuity requirements of the through-thickness stresses.

The classical theory of plates (CTP) [2,3] and its extension to laminated structures, namely classical laminate analysis (CLA) [4], are commonly regarded as inadequate for predicting accurate through-thickness stresses under the conditions described in the previous paragraph. This theory neglects the effects of transverse shear and transverse normal strains, the displacement fields neglect the zigzag effect, and the transverse displacement is assumed to be constant through the thickness.

To overcome these deficiencies a large number of approximate higher-order 2-D theories have been formulated with the aim of predicting accurate 3-D stress fields while maintaining low computational expense. Refinements of CLA along these lines have focused mainly on displacement-based models due to the relatively intuitive physical meaning of the displacement variables that govern the distortion of the plate cross-section. These theories extend from first-order shear deformation theories by Mindlin [5] and Yang, Norris and Stavsky [6] to higher-order Levinson–Reddy-type shear deformation models that enforce vanishing shear strains at the top and bottom surfaces in the displacement field *a priori* [7,8], and further to generalized higher-order theories that do not make this initial assumption and may account for transverse normal deformation, i.e. thickness stretching [9,10]. Finally, starting with the works of Lekhnitskii [11] and Ambartsumyan [12] in the Russian literature, and Di Sciuva [13] and Murakami [14] in the Western literature, attempts were made to incorporate changes in the layerwise slopes of the in-plane displacements u_x and u_y via unknown zigzag bending rotations multiplied by layup-dependent zigzag functions. Since then, more accurate zigzag functions have been proposed by Tessler et al. [15–18] and Icardi [19], with the latter work providing the most recent assessment of different zigzag theories.

A fundamental characteristic of purely displacement-based theories is that all strains and stresses are derived from the displacement assumptions using the kinematic and constitutive equations, respectively, and transverse strains and transverse stresses are typically not recovered accurately in this manner [20]. More accurate transverse stresses can be recovered *a posteriori* by integrating the in-plane stresses in Cauchy's 3-D indefinite equilibrium equations [21], and various techniques exist to achieve this within the displacement-based finite element method (FEM) [22–25]. The disadvantage of this technique is that the post-processed transverse stresses no longer satisfy the underlying equilibrium equations of the theory, in terms of force resultants and moments, and are therefore variationally inconsistent. A second disadvantage of this technique is that higher-order derivatives of the kinematic variables are required, and for C^0-continuous finite elements, computing these derivatives leads to oscillations that require smoothing [22].

The aforementioned post-processing operation can be precluded if independent assumptions for the transverse stresses are made. This results in a mixed displacement/stress-based approach, whereby the governing equilibrium equations and boundary conditions are derived by means of a mixed-variational statement. For example, in the Hellinger–Reissner mixed variational principle [26,27], the strain energy is expressed in complementary form in terms of in-plane and transverse stresses, and Cauchy's 3-D equilibrium equations are introduced as constraints via Lagrange multipliers. This has the advantage that the six stress fields are always equilibrated and provide very accurate predictions of through-thickness stresses [28,29].

Forty years after publishing his work on the Hellinger–Reissner principle, Reissner [30] had the insight that it is sufficient to make separate assumptions for the transverse stresses because only these have to be specified independently to guarantee interfacial continuity requirements. This variational statement is known as Reissner's