# Accepted Manuscript


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PII: S0045-7825(16)31920-X
DOI: http://dx.doi.org/10.1016/j.cma.2017.03.029
Reference: CMA 11386

To appear in: *Comput. Methods Appl. Mech. Engrg.*

Received date : 29 December 2016
Revised date : 21 March 2017
Accepted date : 21 March 2017

Please cite this article as: V. Puzyrev, Q. Deng, V. Calo, Dispersion-optimized quadrature rules for isogeometric analysis: Modified inner products, their dispersion properties, and optimally blended schemes, *Comput. Methods Appl. Mech. Engrg.* (2017), http://dx.doi.org/10.1016/j.cma.2017.03.029

# Dispersion-optimized quadrature rules for isogeometric analysis: modified inner products, their dispersion properties, and optimally blended schemes

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**Abstract**

This paper introduces optimally-blended quadrature rules for isogeometric analysis and analyzes the numerical dispersion of the resulting discretizations. To quantify the approximation errors when we modify the inner products, we generalize the Pythagorean eigenvalue theorem of Strang and Fix. The proposed blended quadrature rules have advantages over alternative integration rules for isogeometric analysis on uniform and non-uniform meshes as well as for different polynomial orders and continuity of the basis. The optimally-blended schemes improve the convergence rate of the method by two orders with respect to the fully-integrated Galerkin method. The proposed technique increases the accuracy and robustness of isogeometric analysis for wave propagation problems.

*Keywords:* Isogeometric analysis, Finite elements, Spectral approximation, Eigenvalue problem, Wave propagation, Numerical dispersion, Quadrature, High order

## 1. Introduction

Dispersion analysis is a powerful tool to understand the approximation errors of a numerical method. Amongst the most popular numerical methods used for wave propagation problems are the finite element and the spectral element methods [28, 44, 45], whose implementations exhibit excellent parallel scalability [17]. The dispersive properties of these methods have been studied in detail to suggest that the most cost-effective scheme can be obtained by an appropriate weighted average of both methods [3, 50, 58]. This idea was employed in [4], where the optimal blending of the finite element and the spectral element methods was obtained and shown to provide two orders of extra accuracy (superconvergence) in the dispersion error. The blended scheme is equivalent to the use of nonstandard quadrature rules and, therefore, it can be efficiently implemented by replacing the standard Gaussian quadrature by a nonstandard rule [3, 4].

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