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Differentiable monotonicity-preserving schemes for discontinuous Galerkin methods on arbitrary meshes

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Abstract

This work is devoted to the design of interior penalty discontinuous Galerkin (dG) schemes that preserve maximum principles at the discrete level for the steady transport and convection-diffusion problems and the respective transient problems with implicit time integration. Monotonic schemes that combine explicit time stepping with dG space discretization are very common, but the design of such schemes for implicit time stepping is rare, and it had only been attained so far for 1D problems. The proposed scheme is based on a piecewise linear dG discretization supplemented with an artificial diffusion that linearly depends on a shock detector that identifies the troublesome areas. In order to define the new shock detector, we have introduced the concept of *discrete local extrema*. The diffusion operator is a graph-Laplacian, instead of the more common finite element discretization of the Laplacian operator, which is essential to keep monotonicity on general meshes and in multi-dimension. The resulting nonlinear stabilization is non-smooth and nonlinear solvers can fail to converge. As a result, we propose a smoothed (twice differentiable) version of the nonlinear stabilization, which allows us to use Newton with line search nonlinear solvers and dramatically improve nonlinear convergence. A theoretical numerical analysis of the proposed schemes show that they satisfy the desired monotonicity properties. Further, the resulting operator is Lipschitz continuous and there exists at least one solution of the discrete problem, even in the non-smooth version. We provide a set of numerical results to support our findings.

Keywords: Finite elements, discrete maximum principle, monotonicity, shock capturing, discontinuous Galerkin, local extrema diminishing.

1. Introduction

The transport problem is one of many problems that might satisfy a maximum principle (MP) or a positivity property. However, its numerical discretization may violate these properties at the discrete level. These violations arise in the form of local spurious oscillations near sharp layers of the solution. Such oscillations break the MP of the continuous problem. For steady problems with no source term, the MP implies that the extrema of the solution are on the boundary of the domain; they are bounded by the boundary and the initial solution extrema in the transient case.

Many authors have focused on developing accurate schemes that inherit the MP at the discrete level, i.e. discrete maximum principle (DMP) preserving schemes. To this end, several approaches have been used. In the case of explicit time integration combined with finite volumes or discontinuous Galerkin (dG) methods, the schemes are usually based on either slope or flux limiters, or special reconstruction algorithms. These methods are widely present in literature and already well understood (see, e.g., [25]).

For implicit time integration and continuous Galerkin (cG) finite element space discretization, methods attaining DMPs are not as well understood as the previous ones. However, several schemes have been developed to date. In this case, most of the approaches are based on adding an artificial diffusion operator. Then, in order to maintain high-order convergence rates in smooth regions, this operator is scaled such that it vanishes in smooth regions and it is active in the vicinity of sharp layers. Depending on how

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