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Higher-order discontinuous Galerkin time stepping and local projection stabilization techniques for the transient Stokes problem

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Abstract

We introduce and analyze discontinuous Galerkin time discretizations coupled with continuous finite element methods based on equal-order interpolation in space for velocity and pressure in transient Stokes problems. Spatial stability of the pressure is ensured by adding a stabilization term based on local projection. We present error estimates for the semi-discrete problem after discretization in space only and for the fully discrete problem. The fully discrete pressure shows an instability in the limit of small time step length. Numerical tests are presented which confirm our theoretical results including the pressure instability.

Keywords: Stabilized finite elements, transient Stokes equations, equal-order elements, local projection, discontinuous Galerkin methods
2000 MSC: 65M12, 65M15, 65M60

1. Introduction

The use of equal-order interpolation for velocity and pressure in incompressible flow problems does not satisfy the inf-sup stability condition and may produce oscillations in the pressure. In order to overcome this difficulty, several stabilization methods have been proposed in the literature. The streamline-upwind Petrov–Galerkin (SUPG) [1–3] and the pressure stabilized Petrov–Galerkin (PSPG) [4] methods are popular tools for the approximation of incompressible flow problems using equal-order interpolation. The common point in stabilized methods is to add extra terms to the discrete formulation in order to enhance the stability of the numerical scheme. The SUPG method combined with the PSPG method has been used to cope the instability of dominating advection due to high Reynolds numbers and the violation of the discrete inf-sup condition of the Navier–Stokes equations. Concerning steady incompressible flow problems, this class of residual based stabilization techniques is still very popular due to its robustness. However, a fundamental drawback of SUPG is that several terms need to be added to the variational form to ensure the strong consistency of the method. Furthermore, the strong coupling between velocity and pressure in the stabilization terms makes the analysis difficult. In order to relax the strong consistency in SUPG and PSPG type methods, several stabilization techniques have been developed. In particular, we mention edge stabilization methods [5, 6], local projection stabilization (LPS) methods as two-level approach [7, 8] or as one-level enrichment method [9], and variational multiscale methods [10].

In the discretization of time-dependent problems, one often uses the methods of lines. In this approach, the problem is discretized in space first whereas the time remains continuous. This methodology leads to a large system of ordinary differential equations which can be solved by a suitable ODE solver. Numerical studies of the SUPG method in space combined with implicit/explicit transient algorithms can be found in [11]. The extension to transient Stokes problems with different stabilization methods including

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