Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 313 (2017) 337–361
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Efficient Markov Chain Monte Carlo for combined Subset Simulation and nonlinear finite element analysis

David K.E. Green

School of Civil & Environmental Engineering, University of New South Wales, Sydney, NSW 2052, Australia

Received 21 March 2016; received in revised form 6 August 2016; accepted 4 October 2016
Available online 12 October 2016

Abstract

Typical probabilistic problems in an engineering context include rare event probability estimation for physical models where spatial autocorrelation of material property parameters is significant. Subset Simulation, a Markov Chain Monte Carlo technique, can be used to estimate rare event probabilities in physical models more efficiently than Monte Carlo Simulation. This efficiency gain is important when the sampling operation is computationally demanding, as is the case in the solution of stochastic Partial Differential Equations. In high dimensional spaces where Polynomial Chaos or other direct integration techniques become intractable, sampling methods may be the only way to compute integral functions in probabilistic analysis. In this paper, Subset Simulation is applied to probability of failure estimation in nonlinear elasto-plastic finite element problems. Further, a derivation of confidence intervals for Subset Simulation relative errors is presented. This new technique allows for vastly improved efficiency in the computation of error estimates for Subset Simulation. Significantly, the numerical studies presented indicate that for the tested finite element problems, Metropolis–Hastings sampling can outperform Componentwise Metropolis–Hastings and Gibbs sampling. This result is relevant to the design of efficient Subset Simulation methodologies.

Keywords: Markov Chain Monte Carlo; Random fields; Subset simulation; Finite elements; Reliability; Probability of failure

1. Introduction

This paper explores the application of Subset Simulation, a sampling based numerical integration technique, to the probabilistic analysis of structural models. Subset Simulation was originally presented in [1] for the estimation of rare event probabilities. Subsequent developments are detailed in [2]. In Civil Engineering, structural problems often have very small probabilities of failure but very large consequences of failure. The efficient estimation of these small probabilities would help to improve risk based estimates of structural safety. A useful definition of risk that can be adopted is probability times the consequence of an event [3]. By quantifying the risk associated with a given design, the level of risk can be assessed based on a comparison with other accepted risks [4]. For risk based analysis of consequential rare events the potential severity of an outcome may still carry high associated risks. In this scenario, it becomes necessary to estimate rare event probabilities for structural systems.

E-mail address: d.k.green@unsw.edu.au.

http://dx.doi.org/10.1016/j.cma.2016.10.012

In soils, for example, spatial autocorrelation of material property parameters is known to have a significant impact on calculated failure probabilities [5]. Probabilistic analyses that aim to quantify risks accurately in such systems must then incorporate the effects of spatial autocorrelation into physical models. Ideally, spatial autocorrelation should be incorporated into existing advanced numerical models, allowing for the developments in probabilistic techniques and general numerical modelling of physical phenomena to occur in parallel. This paper uses autocorrelated models of material property parameters for the physical models discussed.

Popular existing methods for calculating failure probabilities when random fields are used to define the input distribution are the Random Finite Element Method (RFEM) and the variants of the Stochastic Finite Element Method (SFEM). The original RFEM uses a direct Monte Carlo Simulation (MCS) procedure which is discussed further in Section 2.3 of this paper. SFEM and variants such as the Spectral SFEM (SSFEM) [6,7] perform an integration over the random dimensions of the input probability space by expressing this space in terms of orthogonal polynomials (termed the Polynomial Chaos). The disadvantage of the SFEM style approach is that it is maximally inaccurate on the tails of the distribution [8] and as such is a poor candidate for estimating small threshold probabilities very far from the mean response of the model output distribution. Additionally, the size of the probabilistic linear systems that must be solved during an SFEM analysis increases very rapidly with the dimensionality of the problem [6]. The convergence rate of Monte Carlo Simulation, by contrast, is independent of the dimensionality of the problem. Further, elasto-plastic and other nonlinear material models are of interest in engineering practice. Unfortunately, using such models in SFEM type analyses is challenging and remains an active area of research, see [9,10].

For sampling-type solutions of stochastic Partial Differential Equations (PDE), the nonlinearity of constitutive models can be accommodated easily as long as deterministic solutions for that material model are available. RFEM, a direct MCS method [3], is a powerful way of incorporating material property spatial autocorrelation into existing numerical models. RFEM proceeds by first selecting a random field model of material property parameters and/or applied loads, then repeatedly sampling random field realisations from the input distribution, running the deterministic inputs through a finite element solver and then using the output from the discrete deterministic problems to estimate the distribution of outputs. For reliability analysis, RFEM is a very useful approach because no assumptions are made on the output distribution and as such the full system reliability can be calculated regardless of the complexity of the input distribution.

Reliability analysis for Civil Engineering problems often involves the estimation of vanishingly small probabilities of failure [1,2,5]. Direct MCS is theoretically able to compute rare event probabilities, but may require a prohibitively large number of discrete simulations to do so accurately [11,1]. For an RFEM approach, the dimensionality of the probabilistic input space is on the order of the number of elements in a mesh needed to approximate the discrete, deterministic simulations accurately. In the case that nonlinear PDE problems must be solved for each Monte Carlo iteration, the computational cost of random field simulation is negligible compared to the cost of the PDE problems. This issue is discussed in Section 2 of this paper. In this case, the particular choice of random field simulator is not nearly as significant as minimising the number of Monte Carlo iterations. Direct MCS is poorly suited to estimating rare event probabilities associated with numerical approximations to nonlinear PDE systems because the number of iterations cannot be reduced without sacrificing accuracy.

The underlying problem for rare event simulation with combined finite element and random field models is the difficulty of search in high dimensional spaces. As high dimensional spaces are very "large" volumetrically, it can take a long time to find rare portions of the search space by a stochastic search. Human intuition, which is well suited for explorations of one to three dimensional spaces, is poorly adapted to high dimensional search [12]. Further, as the number of probabilistic dimensions increases, it becomes increasingly time consuming to compute quantities of interest accurately and quickly. Techniques that are successful in a small number of dimensions may fail in very high dimensional spaces, as is the case with SFEM. The dimensionality of the numerical problem to be solved in an SFEM type analysis grows by the factorial of the approximation order [8].

To address the small probability of failure problem, Subset Simulation can be applied [1,2]. Subset Simulation, also known as Sequential Monte Carlo, has been applied in areas other than probabilistic engineering mechanics [13,14]. This method is able to calculate small threshold event probabilities more efficiently than direct MCS and is discussed in more detail in Section 2.4. Subset Simulation proceeds by performing a series of Markov Chain Monte Carlo analyses. Essentially, these analyses estimate the probability of some lesser failure condition than the full, desired failure condition. For example, if failure is deemed to occur once displacements of a system exceed some threshold value, a subset failure event would be perhaps one in which displacements reach 80% of the failure threshold