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Elemental enriched spaces for the treatment of weak and strong discontinuous fields

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Abstract

This paper presents a finite element that incorporates weak, strong and both weak plus strong discontinuities with linear interpolations of the unknown jumps for the modeling of internal interfaces. The new enriched space is built by subdividing each triangular or tetrahedral element in several standard linear sub-elements. The new degrees of freedom coming from the assembly of the sub-elements can be eliminated by static condensation at the element level, resulting in two main advantages: first, an elemental enrichment instead of a nodal one, which presents an important reduction of the computing time when the internal interface is moving all around the domain and second, an efficient implementation involving minor modifications allowing to reuse existing finite element codes. The equations for the internal interface are constructed by imposing the local equilibrium between the stresses in the bulk of the element and the tractions driving the cohesive law, with the proper equilibrium operators to account for the linear kinematics of the discontinuity. To improve the continuity of the unknowns on both sides of the elements on which a static condensation is done, a contour integral has been added. These contour integrals named inter-elemental forces can be interpreted as a "do nothing" boundary condition (Coppola-Owen and Codina, 2011) published in another context, or as the usage of weighting functions that ensure convergence of the approach as proposed by J.C. Simo (Simo and Rifai, 1990). A series of numerical tests for scalar unknowns as a simple representation of more general numerical simulations are presented to illustrate the performance of the enriched elemental space.

Keywords: Enriched FE spaces; Internal interfaces; Discontinuous fields; Multi-materials; EFEM; Cracks

1. Introduction

The simultaneous existence of multiple materials with varying physical properties are frequently found in daily life and industrial processes, among many other practical situations. These types of problems are labeled "multi-materials" and they typically exist in different forms depending on the given phase distribution. Examples are

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gas–liquid transport, magma chambers, fluid–fuel interactions, crude oil recovery, spray cans, sediment transport in rivers and floods, pollutant transport in the atmosphere, cloud formation, fuel injection in engines, bubble column reactors and spray dryers for food processing, to name only a few. This demonstrates the great incidence and also the importance of multi-material problems, which probably occur even more frequently than single materials [1]. As a result of the interaction between the different components, multi-materials are rather complex and very difficult to describe theoretically. The design and optimization of multi-material systems therefore requires a deep understanding of the interface transport phenomena. Furthermore, internal moving sharp gradients or discontinuities in the unknown function in homogeneous materials are also problems that need a special treatment similar to the internal interfaces of the multi-material problems. This is for instance the case of crack propagation problems in solids; welding or phase-change in thermal problems; shock waves in compressible flows, among others.

Physical modeling in ad-hoc laboratory scale models is not suitable for this purpose because of its complexity, the difficulty for scaling up to real life problems, its large execution times involved, and hence unaffordable costs and risks. The alternative is nowadays, numerical modeling. For homogeneous flows, computational fluid dynamics (CFD) has already a long history and it is standard a practice to use commercially available CFD codes for the design of, for instance, airplanes and cars. However, because of the complex physics involved in multi-fluid flows the application of CFD in this area is rather young (probably 20–30 years). Despite the practical importance of the problem and the intensive work carried out in the last decade for the development of suitable mathematical and computational models, it is widely accepted that the numerical study of heterogeneous flows is still a major challenge [1]. Different reasons exist for this fact; some are connected to the complex mathematical structure of the multi-fluid problem, others are related to the multi-scale features of the flow. For instance, the presence of breaking waves in a free surface, the existence of one or multiple internal interfaces and, in general, the high unsteadiness of the flow, constitute major obstacles for the analysis. Adding to this the need to reproduce the interaction of free-surface flows with structures as it occurs in many practical problems, may be clearly understood as the reason of why multi-fluid flow problems represent nowadays one of the great challenges in computational engineering science.

In the case of multi-materials, the dynamics of the interface between fluids plays a dominant role. The computation of the interface between various immiscible fluids or the free surfaces is extremely difficult because neither the shape nor the positions of the interfaces are a priori known. The approaches to solve these problems are mainly two: one is based on using a moving mesh that follows the discontinuity, named interface-tracking methods, and the second based on using a fixed mesh refined in that part of the domain where the interface cross during the evaluation named interface-capturing methods. The former computes the motion of the flow particles via a Lagrangian approach where the computational domain adapts itself to the shape and position of the interfaces (see e.g., [2–7]). A different approach for the simulation of free-surface flows that is based on Lagrangian particles can be found in [8–11].

In the interface-capturing method (see [12–14]), the interface is represented by a surface mesh advected with a Lagrangian method while immersed in a Eulerian (fix) mesh where the flow problem is solved considering the fluids as a single effective fluid with variable properties. Popular methods of this type are the volume-of-fluid technique (see [15–17]) and the level set method (see, for example, [18–21]). In this case, the flow problem is also solved in a fixed underlying mesh considering a single fluid with variable properties. Variants of these methods mainly differ in two aspects: first, the technique used to solve the transport equation for the scalar function where the interface is embedded, for which a great deal of work has been carried out to improve the accuracy for purely Eulerian methods [22–27] and for semi-Lagrangian methods [28–30]. Second, the technique used to solve the Navier–Stokes equations for a one-phase flow with variable properties and in how the fluid-dynamics variables are treated near the interface because these can exhibit discontinuities in their values and/or their gradients owing to the discontinuities in the physical properties and/or the presence of singular forces. Several remedies have been proposed to improve accuracy and robustness of computations in Eulerian formulations. For instance, in Brackbill et al. [31], a treatment of the singular forces at the interface by means of a regularization is proposed, such that sharp variations in the pressure field are avoided. In Löhner et al. [32] and Carrica et al. [33], different extrapolation techniques of the velocity and pressure near the interface are presented.

The finite element solution, either continuous or discontinuous across inter-element boundaries, for such problems when the interface does not necessarily conform to the element edges (in 2D) or faces (in 3D), suffer of sub-optimal approximation orders. This poor approximation leads to spurious velocities near the interface that may significantly affect the precision and the robustness of numerical simulations (see e.g., [34]). A number of methods have been developed to overcome these difficulties. One possibility is to locally modify the finite element spaces in