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An overlapping Domain Decomposition preconditioning method for monolithic solution of shear bands

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Abstract

Metals subjected to high strain rate impact often exhibit a sudden and profound drop in the material's load bearing capability, a ductile failure phenomenon known as shear banding. Shear bands, characterized as material instabilities, are driven by shear heating and described as narrow regions that have sustained intense plastic deformation and high temperature rise. This coupled thermo-mechanical localization problem can be formulated as a nonlinear system with two balance equations, momentum and energy, and two constitutive laws for elasticity and plasticity.

In our formulation, mixed finite elements are used to discretize the equations in space and an implicit finite difference scheme is used to advance the system in time, where at every time step, a Newton type method is used to solve the system monolithically. To that end, a block Jacobian matrix is formed analytically using Gâteaux derivatives. The resulting Jacobian is sparse, nonsymmetric and its sparsity pattern and eigenvalue content vary with the different stages of shear bands formation, consequently posing a significant challenge to iterative solvers.

To address this issue, an overlapping Domain Decomposition preconditioner that takes into account the physics of shear bands and the domain in which it forms, is proposed. The key idea is to resolve the shear band domain accurately while only solving the remaining domain approximately, with selective updates when necessary. The preconditioner is formed on the basis of an additive Schwartz method that is applied to a Schur complement partition of the system.

The proposed preconditioner is implemented in parallel and tested on three different benchmark problems as compared against off-the-shelf solvers. We study the effect of h- and k-refinement that are obtained from Isogeometric discretizations of the system, the overlapping strategy and its parallel performance. Excellent performance is demonstrated in serial and parallel on all benchmark examples.
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Keywords: Shear bands; Preconditioner; Multiphysics; Schur complement; Domain Decomposition; Parallel scalability

1. Introduction and problem statement

Shear banding is the localization of plastic strain into narrow bands that is typically observed in metals (and other materials) undergoing high strain rate loading. Shear bands, characterized in the literature as material instabilities [1],

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are driven by shear heating and result in the material's reduced stress bearing capacity and fracture. Accurate modeling of shear bands is very challenging due to the complicated multiphysics and difficult numerical modeling it requires. The problem is a highly nonlinear, coupled thermo-mechanical problem with localized crack-like domains that require specialized discretizations and solvers.

Experimentally derived material models for shear bands describe plastic flow as being dependent on temperature, strain rate, and a hardening parameter [2]. While several models are available, all are similar in that increasing temperature (due to plastic work) has a softening effect, causing plastic flow to occur more readily, while increases in strain rate and the hardening parameter have a hardening effect.

The experimental work of [3] shows that shear bands form in three stages. In Stage 1 the material deforms homogeneously and the temperature rises uniformly. Stage 2 marks the onset of shear band localization with a localized temperature rise leading to strain softening effect. Finally, in Stage 3, severe localization and rapid softening occurs, a phenomenon known as stress-collapse, which indicates a sudden and large drop in the material's load bearing capability. For more discussion on the physics of shear bands, the reader is referred to the reviews in [4–6].

Shear bands can be modeled by a set of four nonlinear partial differential equations (PDEs) [2,7,8], two of which are balance equations: conservation of momentum and thermal energy, and two elastic and plastic constitutive equations. Under the assumption of small deformations, these four coupled equations, written in tensorial notation, are

$$\begin{cases} \rho \ddot{\underline{u}} &= \underline{\nabla} \cdot \underline{\underline{\sigma}} \\ \rho c \dot{T} &= \kappa \underline{\nabla} \cdot \left(\underline{\nabla}\, T \right) + \chi \bar{\sigma} g(\bar{\sigma}, T, \bar{\gamma}_p) \\ \dot{\underline{\underline{\sigma}}} &= \underline{\underline{\underline{\underline{C}}}}^{elas} : \left(\underline{\nabla}^s \dot{\underline{u}} - \dfrac{3}{2} \dfrac{g(\bar{\sigma}, T, \bar{\gamma}_p)}{\bar{\sigma}} \underline{\underline{S}} - \alpha \dot{T} \underline{\underline{I}} \right) \\ \dot{\bar{\gamma}}_p &= g(\bar{\sigma}, T, \bar{\gamma}_p) \end{cases} \quad \text{in } \Omega \tag{1}$$

where one, two and four underlines indicate first-, second- and fourth-order tensors respectively; a colon (double dot) indicates the double contraction operator for tensors; an overline indicates an equivalent quantity; $\underline{\nabla}\, \bullet$ and $\underline{\nabla} \cdot \bullet$ are the gradient and the divergence operators and $\underline{\nabla}^s \bullet$ is the symmetric part of the gradient operator, that is $\underline{\nabla}^s \bullet = \frac{1}{2}\left(\underline{\nabla}\, \bullet + (\underline{\nabla}\, \bullet)^T\right)$. The four unknown variables are the displacement field $\underline{u}$, the temperature field T, the stress field $\underline{\underline{\sigma}}$ and the equivalent plastic strain (EQPS) field $\bar{\gamma}_p$. $\underline{\underline{S}}$ is the deviatoric part of $\underline{\underline{\sigma}}$ defined as $\underline{\underline{S}} = \underline{\underline{\sigma}} - \frac{tr(\underline{\underline{\sigma}})}{3} \underline{\underline{I}}$ and $\bar{\sigma}$ is the von Mises equivalent stress defined as $\bar{\sigma} = \sqrt{\frac{3}{2} \underline{\underline{S}} : \underline{\underline{S}}}$ (see [9]); $\underline{\underline{I}}$ is the second order identity tensor. The material parameters in the model are: ρ the density, c the specific heat, κ the thermal conductivity, χ the Taylor–Quinney coefficient which defines the amount of plastic work converted to heat, α the thermal expansion coefficient, $\underline{\underline{\underline{\underline{C}}}}^{elas}$ the elastic fourth order tensor and the flow law g that depends on the equivalent stress, the equivalent plastic strain and the temperature. An implicit expression for g can be written by assuming that strain hardening, strain rate hardening and thermal softening effects are decomposed multiplicatively as follows

$$\frac{\bar{\sigma}}{\sigma_{ref}} = P(T) Q(\bar{\gamma}_p) R(\dot{\bar{\gamma}}_p) \tag{2}$$

where σ_{ref} is a reference stress flow. Rearranging the terms in (2) and assuming that R is invertible, one can write

$$g(\bar{\sigma}, T, \bar{\gamma}_p) = \dot{\bar{\gamma}}_p = R^{-1} \left(\frac{\bar{\sigma}/\sigma_{ref}}{P(T) Q(\bar{\gamma}_p)} \right) \tag{3}$$

Ω is the problem domain and its boundary $\Gamma = \Gamma_d \cup \Gamma_n$ and $\Gamma_n \cap \Gamma_d = \emptyset$, where Γ_d and Γ_n are the part of the boundary where Dirichlet and Neumann conditions are prescribed, respectively. The boundary conditions applied to the problem are as follows

$$\begin{aligned} &\underline{u}(\underline{x}, t) = \bar{\underline{u}}(\underline{x}, t) && \text{on } \Gamma_d^u, \\ &\underline{n}.\underline{\underline{\sigma}}(\underline{x}, t) = \bar{\underline{t}}(\underline{x}, t) && \text{on } \Gamma_n^u, \\ &T(\underline{x}, t) = \bar{T}(\underline{x}, t) && \text{on } \Gamma_d^T, \\ &\underline{n}.\underline{q}(\underline{x}, t) = \bar{q}(\underline{x}, t) && \text{on } \Gamma_n^T \end{aligned} \tag{4}$$

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