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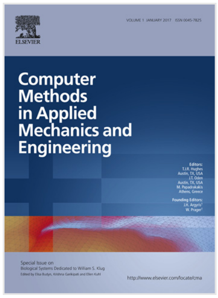

A tunable finite difference method for fractional differential equations with non-smooth solutions

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A tunable finite difference method for fractional differential equations with non-smooth solutions

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Abstract

In this work, a finite difference method of tunable accuracy for fractional differential equations (FDEs) with end-point singularities is developed. Modified weighted shifted Grünwald–Letnikov (WSGL) formulas are proposed to approximate the left and right Riemann–Liouville fractional operators, which show better accuracy than the original WSGL formulas, due to the use of the correction terms. Finite difference schemes are constructed to solve two fractional boundary value problems and a space-fractional Allen–Cahn equation. Even if the singularity of the considered FDEs is unknown, satisfactory numerical solutions can still be obtained by suitably tuning the correction terms.Various numerical examples are presented to verify the effectiveness of the present method, and comparisons with other known methods are also made that demonstrate higher accuracy of the current method.

Keywords:
Weighted shifted Grünwald–Letnikov formula, correction terms, fractional boundary value problems, space-fractional Allen–Cahn equation, singularities.

1. Introduction

This paper aims to provide a numerical method with tunable accuracy to solve the FDEs with end-point singularities. Because the fractional operators are nonlocal with weakly singular kernels, the fractional models are more complicated than the classical models. Hence, the solutions of fractional models are also more complicated and they are often non-smooth. There are several analytical methods to solve fractional differential equations, such as the Fourier transform method, the Laplace transform method, the Mellin transform method, the Green's function method, and so on [1–5]. However, analytical methods do not work well on most of FDEs, e.g. with non-constant coefficients or nonlinearities. Hence, developing numerical methods is of great importance in practical applications.

There have been various numerical methods to solve FDEs, such as finite difference methods [6–13], finite element methods [14–19], and spectral methods [20–26]. However, most of the known

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