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An adaptive, hanging-node, discontinuous isogeometric analysis method for the first-order form of the neutron transport equation with discrete ordinate (S_N) angular discretisation

A.R. Owens*, J.A. Welch, J. Kópházi, M.D. Eaton

Nuclear Engineering Group, Department of Mechanical Engineering, City and Guilds Building, Imperial College London, Exhibition Road, South Kensington, London, SW7 2AZ, United Kingdom

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Highlights

- A DG hanging-node IGA discretisation is developed for hyperbolic equations.
- A conservative upwinding scheme across the curved element boundaries is established.
- Optimal convergence rates are achieved when the underlying solution is smooth.
- Adaptive formulation outperforms uniform refinement for a variety of test cases.
- Comparisons made to DG finite elements.

Abstract

In this paper a discontinuous, hanging-node, isogeometric analysis (IGA) method is developed and applied to the first-order form of the neutron transport equation with a discrete ordinate (S_N) angular discretisation in two-dimensional space. The complexities involved in upwinding across curved element boundaries that contain hanging-nodes have been addressed to ensure that the scheme remains conservative. A robust algorithm for cycle-breaking has also been introduced in order to develop a unique sweep ordering of the elements for each discrete ordinates direction. The convergence rate of the scheme has been verified using the method of manufactured solutions (MMS) with a smooth solution. Heuristic error indicators have been used to drive an adaptive mesh refinement (AMR) algorithm to take advantage of the hanging-node discretisation. The effectiveness of this method is demonstrated for three test cases. The first is a homogeneous square in a vacuum with varying mean free path and a prescribed extraneous unit source. The second test case is a radiation shielding problem and the third is a 3×3 "supercell" featuring a burnable absorber. In

* Corresponding author.
E-mail address: a.owens12@imperial.ac.uk (A.R. Owens).

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the final test case, comparisons are made to the discontinuous Galerkin finite element method (DGFEM) using both straight-sided and curved quadratic finite elements.

Keywords: Adaptive; Hanging-node; Isogeometric analysis; Hyperbolic equations; Neutron transport; Discrete ordinates

1. Introduction

The numerical solution of the neutron transport equation for heterogeneous whole core reactor physics and radiation shielding problems remains a significant computational challenge even for today's numerical algorithms and high performance computing hardware architectures. This is primarily due to the seven-dimensional phase space of the problem, with solution fidelity required in energy, direction and space, as well as time in transient problems. Current numerical methods deployed in reactor physics involve first solving the neutron transport equation for fuel assembly sized domains with prescribed periodic boundary conditions [1,2]. The heterogeneous material cross section data for the fuel assembly is then homogenised for use in whole core reactor physics problems. The whole core reactor physics problem is then often solved using the neutron diffusion equation with the assembly averaged or homogenised material cross section data by applying nodal discretisation methods [1,3].

The discontinuous Galerkin finite element method (DGFEM) is a commonly used approach if the geometric features are to be modelled directly. DGFEM was originally developed and applied to the discretisation of the first-order form of the neutron transport equation with a discrete ordinates angular discretisation [4,5]. It has since been applied to a wide variety of hyperbolic systems, such as the compressible Navier–Stokes equations [6], the Euler equations of gas dynamics [7] and radiative heat transfer [8].

A wide range of complex geometries can be modelled in the finite element method using basic geometric primitives such as triangles and quadrilaterals in two dimensions and pyramids, prisms, hexahedra and tetrahedra in three dimensions [9]. This is achieved by the use of automatic mesh generation software that generates surface and volumetric meshes of finite elements from the underlying Non-Uniform Rational B-Spline (NURBS) surface descriptions produced by Computer Aided Design (CAD) programs [9]. However, the underlying geometry is often not exactly reproduced when it is discretised into basic geometric primitives as these usually are linear elements with straight or planar sides [9]. This is of particular importance in reactor physics applications, where the fissile mass of the system must be preserved by the polygonal approximations to the geometry in order to obtain accurate criticality solutions [10,11]. This can be corrected by locally modifying the mesh to preserve the fissile mass, but this is a non-trivial task for complex geometries.

Recently, the finite element method (FEM) has been extended to allow exact preservation of the CAD geometry using isogeometric analysis [12]. As in FEM, the weak form of the governing partial differential equation (PDE) is discretised using prescribed shape and test functions to form a system of equations. To preserve the CAD geometry, the same NURBS used in the CAD program are used as the shape and test functions in the spatial discretisation of the PDE. This allows the geometry to be modelled exactly from the coarsest level of refinement, as well as maintaining this exact representation as the mesh is refined.

Since its introduction IGA has been applied to a wide variety of different physical phenomena, including computational fluid dynamics [12], computational solid mechanics [12] and coupled solid–fluid interaction problems [13]. A variety of different IGA discretisations have been developed. These were initially restricted to Bubnov–Galerkin type IGA discretisations for solid mechanics problems and streamline upwind Petrov–Galerkin discretisations for advection–diffusion problems [12]. In the field of reactor analysis, Hall et al. applied a Bubnov–Galerkin projection using NURBS to the one group neutron diffusion equation for a simple pincell problem [14]. This was extended by Welch et al. to a multigroup scheme for large heterogeneous quarter-core reactor physics problems [15].

Discontinuous IGA methods have recently been developed for elliptic systems of equations [16,17]. For hyperbolic systems, two discontinuous IGA discretisations have recently been developed. The Blended Isogeometric Discontinuous Galerkin method [18] uses rational Bernstein–Bézier triangles to mesh the geometry. The authors then used standard polynomial spaces to approximate solution fields to the acoustic wave equations and Maxwell's