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Bayesian system identification based on hierarchical sparse Bayesian learning and Gibbs sampling with application to structural damage assessment

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Highlights

- Two Gibbs Sampling algorithms based on noisy incomplete modal data for Bayesian system identification are proposed.
- Spatially-sparse stiffness changes are imposed to alleviate the ill-conditioning when updating a structural model.
- It is not required to solve a nonlinear eigenvalue problem of a structural model.
- The equation-error precision parameter is marginalized over analytically to reduce the posterior sample variances.
- The IASE-ASCE Phase II simulated and experimental benchmark data are utilized to illustrate application of the algorithms.

Abstract

Bayesian system identification has attracted substantial interest in recent years for inferring structural models based on measured dynamic response from a structural dynamical system. The focus in this paper is Bayesian system identification based on noisy incomplete modal data where we can impose spatially-sparse stiffness changes when updating a structural model. To this end, based on a similar hierarchical sparse Bayesian learning model from our previous work, we propose two Gibbs sampling algorithms. The algorithms differ in their strategies to deal with the posterior uncertainty of the equation-error precision parameter, but both sample from the conditional posterior probability density functions (PDFs) for the structural stiffness parameters and system modal parameters. The effective dimension for the Gibbs sampling is low because iterative sampling is done from only three conditional posterior PDFs that correspond to three parameter groups, along with sampling of the equation-error precision parameter from another conditional posterior PDF in one of the algorithms where it is not integrated out as a “nuisance” parameter. A nice feature from a computational perspective is that it is not necessary to solve a nonlinear eigenvalue problem of a structural model. The effectiveness and robustness of the proposed algorithms are illustrated by applying them to the IASE-ASCE Phase II simulated and experimental benchmark studies. The goal is to use incomplete modal data identified before and after possible damage to detect and assess spatially-sparse stiffness reductions induced by any damage. Our past and current focus on meeting challenges arising from Bayesian inference of structural stiffness serves to strengthen the capability of vibration-based structural system identification

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but our methods also have much broader applicability for inverse problems in science and technology where system matrices are to be inferred from noisy partial information about their eigenquantities.

Keywords: Bayesian system identification; Sparse Bayesian learning; Hierarchical model; Gibbs sampling; Damage assessment; IASE-ASCE Phase II benchmark

Nomenclature

N_m = Number of extracted modes in the modal identification we use ($i = 1, \ldots, N_m$)
N_o = Number of measured degrees of freedom
N_d = Number of degrees of freedom of the identification model we use ($k = 1, \ldots, N_d$)
N_s = Number of time segments of measured modal data we use ($r = 1, \ldots, N_s$)
N_θ = Number of substructures considered we use ($j = 1, \ldots, N_\theta$)
$\mathbf{M}, \mathbf{K}$ = Mass and stiffness matrices of the structural model for identification
$\boldsymbol{\theta}$ = Structural stiffness scaling parameters
$\boldsymbol{\theta}_u$ = Structural stiffness scaling parameters for undamaged calibration stages
$\boldsymbol{\phi}_r, \boldsymbol{\omega}_r^2$ = System mode shape and system natural frequencies of the rth mode
β = Equation-error precision parameter
$\widehat{\boldsymbol{\omega}}_{r,i}^2, \widehat{\boldsymbol{\psi}}_{r,i}$ = MAP estimates of modal frequency and mode shape of ith mode from the rth data segment from modal identification
$\boldsymbol{\Gamma}$ = Matrix that picks the measured degrees of freedom from the system mode shape
$\mathbf{T}$ = Matrix relating the vector of N_s sets of identified natural frequencies $\widehat{\boldsymbol{\omega}}^2$ and the system natural frequencies $\boldsymbol{\omega}^2$
$\boldsymbol{\alpha}$ = Variance parameter for the likelihood function of structural stiffness scaling parameters $\boldsymbol{\theta}$
η, ρ = Measurement-error variance parameters for system mode shapes and natural frequencies.

1. Introduction

Inverse problems are a core part of structural system identification, which is concerned with the determination of structural models and their parameters (e.g., stiffness) based on measured structural dynamic response (e.g., [1–5]). However, real inverse problems when treated deterministically are typically ill-conditioned and often ill-posed when using noisy incomplete data, i.e., uniqueness, existence and robustness to noise of an inverse solution is not guaranteed. To deal with ill-conditioning and ill-posedness in inverse problems, a common approach is to avoid an explicit treatment of uncertainty and employ a least-squares approach with the addition of a regularization term to the data-matching term in the objective function to be optimized, often called Tikhonov Regularization (e.g., [6,7]). A regularized least-squares approach usually leads to a well-conditioned and well-posed deterministic optimization problem; however, the relationship of the unique solution to an exact solution of the original inverse problem is uncertain.

The values of parameters of structural models used to predict structural behavior are uncertain. Indeed, since these models are based on simplifying and approximate assumptions, there are really no true values of the model parameters. These modeling uncertainties suggest that when solving inverse problems, we should not just search for a single “optimal” parameter vector to specify the structural model, but rather attempt to describe the family of all plausible values of the model parameter vector that are consistent with both the observations and our prior information. This leads us to consider inverse problems in structural system identification from a full Bayesian perspective, which provides a robust and rigorous framework due to its ability to account for model uncertainties [1,8–11]. The posterior probability distribution of the model parameters from Bayes Theorem is used to quantify the plausibility of all models, within a specified set of models, based on the available data. As an aside, we note that any Tikhonov-regularized solution can always be viewed as the MAP (maximum a posteriori) solution of a Bayesian inversion where the log likelihood