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Bloch theorem with revised boundary conditions applied to glide, screw and rotational symmetric structures

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Highlights

- Quasi-two dimensional wave propagation in periodic structures is investigated.
- Glide, screw and rotational periods are used as unit cells in the Bloch theorem.
- The boundary conditions are revisited keeping the Cartesian coordinate system.
- The eigenvalue problem is reduced and resulting dispersion curves are more clear.
- Screw symmetric structures that do not possess pure translation are analyzed.

Abstract

Wave propagation in complex periodic systems is often addressed with the Bloch theorem, and consists in applying periodic boundary conditions to a discretized unit cell. While this method has been developed for structures periodic by translation, in a recent work, for quasi-one-dimensional wave propagation, it has been shown that screw (translation plus rotation) and glide (translation plus reflection) periodicities can be accounted for as well, keeping the Cartesian coordinate system but revisiting the periodic boundary conditions. The goal of the present paper is to generalize this concept to quasi-two-dimensional wave propagation (two dimensional waves propagating in three dimensional structures). Dispersion relations for a set of reduced problems are then compared to results from the classical method, when available. By considering a smaller periodicity, the computational cost is decreased and the number of folding curves and non-interacting intersecting curves is reduced, improving their interpretability. While the size of a unit cell is divided by a factor two when glide symmetries are considered, this ratio is significantly increased for screw or rotational symmetries. Moreover, the proposed revisited Bloch method is applicable to screw symmetric structures that do not possess purely translational symmetries, and for which the classical method cannot be used (e.g. chiral nanotubes, longitudinally wrinkled helicoids).

Keywords: Bloch theorem; Periodic boundary conditions; Glide, screw and rotational symmetry; Quasi-two-dimensional wave propagation; Dispersion analysis

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1. Introduction

Wave propagation in infinite periodic structures is of interest in a large range of applications such as acoustic metamaterials [1,2] and non-destructive evaluation for structural health monitoring [3]. For complex structures, these challenges are often being addressed via the Bloch theorem, restricting the analysis to a single unit cell to which periodic boundary conditions are applied. For discretized unit cells, Bloch theorem can be implemented following two different approaches: the transfer-matrix method [4–6] and the inverse method [7–10] (see a review of both methods in [11]). Contrary to the inverse approach, the transfer-matrix method has the advantage of providing evanescent modes, but its implementation in multiple dimensions requires more efforts [12,13]. Besides the Bloch theorem, other approaches can be used to investigate wave propagation in infinite periodic structures, such as for example the homogenization methods for low [14,15] and moderated [16] frequencies. However, this paper focuses on the use of the Bloch theorem.

While the Bloch theorem applies to structures periodic by translation, it has been recently shown that for quasi-one-dimensional wave propagation, glide (translation plus reflection) and screw (translation plus rotation) symmetries can be accounted for as well with both the transfer-matrix and the inverse approaches [17]. Advantages are that the considered unit cell is smaller, reducing the computational cost and error, while improving the dispersion curve readability. By quasi-one-dimensional wave propagation, we mean that the wave propagates in one direction, whereas the structure is at least two dimensional (the second dimension being finite and small). The goal of the present paper is to extend this concept to quasi-two-dimensional wave propagation: two-dimensional waves propagating in three-dimensional structures.

However, in quasi-two dimensions, some restrictions on the symmetries exist, which are presented in Table 1. For quasi-one-dimensional structures, while translated, a unit cell can be rotated (screw) or reflected (glide) by an angle θ and the periodicity is along the translational axis. Since these transformations apply around an axis, the periodicity cannot be extended to a second straight direction, except for one specific case, a glide symmetry with a plane of reflection coinciding with the two directions of periodicity ($\theta = 0$). To get a screw symmetric structure with two directions of periodicity, the circumferential direction has to be considered. Indeed, in some specific case as a helix, the structure can be parametrized from two different directions: along the helix axis, or along the circumferential direction, and these two possibilities are respectively referred to as t^* and θ^* in Table 1. This justifies also the presence of rotational symmetries which are screw symmetries without translational components.

These symmetries are illustrated in Fig. 1. Quasi-one-dimensional wave propagation in glide symmetric undulated or buckled beams is investigated in [18–21] (Fig. 1a), and accounting for the glide divides the size of the problem by a factor two [17]. Twisted cables (Fig. 1b) are good examples for which computation gains are even more important. Indeed, considering the classical Bloch theorem, the period by translation is a helix step (2π), whereas with appropriate boundary conditions, the height of the unit cell can be arbitrarily reduced to the length of one finite element [17,22]. Otherwise, the cable slice can be defined with an infinitesimally small thickness, leading to the semi-analytical finite element (SAFE) method [23,24]. Another interesting example is the tetrahelix (Fig. 1c) built by stacking tetrahedra, such that two consecutive tetrahedra are shifted by an irrational angle: there is no period by pure translation. Consequently, it is not possible to apply the classical Bloch theorem, whereas the screw symmetry can be used [17]. The same holds for longitudinally wrinkled helicoids [25–27], and is detailed in Section 3.1.1.

Examples of structures possessing one translational periodicity in one direction and one glide plane symmetry in a second direction are the undulated metal sheet commonly used as materials for roofs (Fig. 1d) and some corrugated cores as the one shown in Fig. 1e. Structures with double-glide periodicity can be found also in corrugated cores as the pyramidal one (Fig. 1f and Section 3.2.2), and in composite and fabric materials (e.g. the plane weave pattern, Fig. 1g). An example of quasi-two-dimensional wave propagation with one wave propagating along the circumferential direction is the cylindrically curved metamaterial panel shown in Fig. 1h [28]. In this last work, Bloch theorem is used to get the dispersion characteristics, discretizing the unit cell in the cylindrical coordinate system. This differs from the present approach where the Cartesian coordinate system is used, providing a simplified formulation and a natural choice for computer implementation. Other examples with rotations are the screw symmetric pipes (Fig. 1i) and the chiral nanotubes [29] investigated in Section 3.2.4.

This paper completes the initial work on quasi-one-dimensional wave propagation [17], and is organized as follows. In Section 2, appropriate boundary conditions are proposed to account for the additional symmetries for the inverse method of the Bloch theorem. In Section 3, the applicability of the proposed method is demonstrated in a set of problems, followed by the conclusions.