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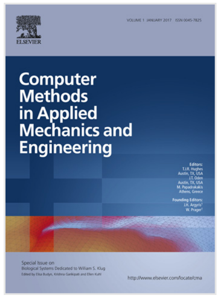

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The optimal multilevel Monte-Carlo approximation of the stochastic drift-diffusion-Poisson system

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Abstract

Existence and local-uniqueness theorems for weak solutions of a system consisting of the drift-diffusion-Poisson equations and the Poisson-Boltzmann equation, all with stochastic coefficients, are presented. For the numerical approximation of the expected value of the solution of the system, we develop a multi-level Monte-Carlo (MLMC) finite-element method (FEM) and we analyze its rate of convergence and its computational complexity. This allows to find the optimal choice of discretization parameters. Finally, numerical results show the efficiency of the method. Applications are, among others, noise and fluctuations in nanoscale transistors, in field-effect bio- and gas sensors, and in nanopores.

Keywords: Stochastic drift-diffusion-Poisson system, existence and uniqueness, multi-level Monte-Carlo finite-element method, optimal method.

1. Introduction

In this work, we consider the system consisting of the drift-diffusion-Poisson equations coupled with the Poisson-Boltzmann equation, all with random coefficients. We show existence and local uniqueness of weak solutions for the stationary problem. This system is a general model for transport processes, where a stochastic process determines the coefficients. Furthermore, we develop a multi-level (ML) Monte-Carlo (MC) finite-element method (FEM) for the system of equations. The different types of errors in the numerical approximation must be balanced and the optimal approach is found here.

In the system of equations considered here, both the operators and the forcing terms are stochastic, and therefore this system has numerous applications (see Figure 1). A deterministic and simplified version, without the Poisson-Boltzmann equation, is the standard model for semiconductor devices. Nowadays, randomness due to the location of impurity atoms is the most important effect limiting the design of integrated circuits. This application area is included in the present model equations. Furthermore, the full system of equations considered here describes a very general class of field-effect sensors including their most recent incarnation, nanowire bio- and gas sensors. While previous mathematical modeling has focused on the deterministic problem and stochastic surface reactions [1, 2, 3, 4, 5, 6], the present model describes how various stochastic processes propagate through a PDE model and result in noise and

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