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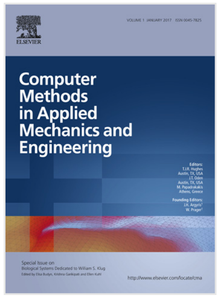

The use of the local truncation error for the increase in accuracy of the linear finite elements for heat transfer problems

A. Idesman, B. Dey

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The use of the local truncation error for the increase in accuracy of the linear finite elements for heat transfer problems.

A. Idesman*, B. Dey

Department of Mechanical Engineering, Texas Tech University, Lubbock, TX 79409-1021, USA

Abstract

A new approach for the increase in the order of accuracy of the linear finite elements used for the time dependent heat equation and for the time independent Laplace equation has been suggested. It is based on the optimization of the coefficients of the corresponding discrete stencil equation with respect to the local truncation error. By a simple modification of the coefficients of the elemental mass and stiffness matrices, the accuracy of the linear finite elements is improved by two orders for the heat equation and by four orders for the Laplace equation. Despite the significant increase in accuracy, the computational costs of the new technique are the same as those for the conventional linear finite elements on a given mesh. The 1-*D*, 2-*D* and 3-*D* numerical examples are in a good agreement with the theoretical results for the new approach and also show that the new linear finite elements are much more accurate than the conventional linear and quadratic finite elements at the same numbers of degrees of freedom.

Keywords: heat equation, Laplace equation, linear finite elements, local truncation error, increase in the order of accuracy

1. Introduction

Heat transfer in an isotropic homogeneous medium is described by the heat equation in domain Ω:

$$\frac{\partial u}{\partial t} - a\nabla^2 u = f \tag{1}$$

for transient problems and by the Laplace equation:

$$\nabla^2 u = f \tag{2}$$

for steady-state problems with the boundary conditions $\boldsymbol{n} \cdot \nabla u = g_1$ on Γ^t and $u = g_2$ on Γ^u, and the initial conditions $u(\boldsymbol{x}, t = 0) = g_3$ in Ω (the initial conditions are given for the transient problems only). Here, u is the temperature, f is a given loading function that may depend on the space coordinate and time for the transient problems and on the space coordinate for the Laplace equation, ∇^2 is the Laplace operator ($\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in the 2-*D* case and $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ in the 3-*D* case), a is the thermal diffusivity (e.g., for homogeneous isotropic materials $a = k/(c\rho)$ where k is the conductivity coefficient, ρ is the density and c is the capacitance), t is the time, Γ^t and Γ^u denote the natural and essential boundaries, g_i ($i = 1, 2, 3$) are

*Corresponding author. Tel.: +1 806 834 2606, fax: +1 806 742 3540.
Email address: alexander.idesman@ttu.edu (A. Idesman*)