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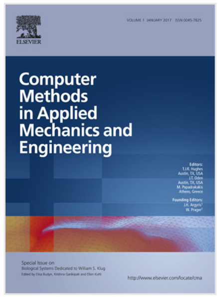

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Stabilized mixed displacement-pressure finite element formulation for linear hydrodynamic problems with free surfaces

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Abstract

A mixed displacement-pressure formulation of the Stokes problem for incompressible fluids with free surfaces is developed for modeling the propagation of gravity waves in liquids and their interaction with structures using a Lagrangian approach. We assume that fluid displacements are small, making convective effects negligible and approximate the fluid velocities from the time derivative of the displacements. The resulting finite element equations are discretized with equal order for both displacement and pressure terms, together with employing stabilization techniques that circumvent the *inf-sup* requirements. The stability and accuracy of the methodology is finally demonstrated by solving some classical problems of hydrodynamics with free surfaces, comparing the results with known analytical solutions.

Keywords: Stokes flow, Displacement formulation, Stabilization, Gravity waves

1. Introduction

The simulation of incompressible fluid flow problems with free surfaces has important applications in many fields of science and engineering, e.g., study of sloshing phenomena in fluid tanks, evaluation of forces due to wave-body interaction, hydrodynamics of large floating structures, propagation of large water waves, etc. In general, the incompressible flow of viscous fluids is governed by the Navier-Stokes equations but, in many applications or particular situations, the convective terms can be neglected and the equations are simplified to the Stokes equations with gravity body forces, eliminating the strong non-linearity associated to convection. The dependent variables in both cases are the velocity and the pressure of the fluid expressed in an Eulerian frame of reference.

Two well known instabilities are associated to the solution of incompressible Navier-Stokes equations using mixed velocity-pressure formulations combined with a standard Galerkin finite element approximation. First, we have the instabilities that appear in convective dominated flows. These instabilities can be palliated by adding some extra dissipation in the flow direction, for example using the stream upwind stabilization Petrov-Galerkin method (SUPG) proposed by Hughes et. al. in [1]. Second, there is the need to satisfy an affinity condition between the velocity and pressure approximation spaces, known as the Ladyzhenskaya-Babuška-Brezzi (LBB) condition or

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