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Gradient damage *vs* phase-field approaches for fracture: Similarities and differences

René de Borst[a,*], Clemens V. Verhoosel[b]

[a] *University of Sheffield, Department of Civil and Structural Engineering, Mappin Street, Sir Frederick Mappin Building, Sheffield S1 3JD, UK*
[b] *Eindhoven University of Technology, Department of Mechanical Engineering, P.O. Box 513, 5600MB Eindhoven, Netherlands*

Highlights

- The mathematical structure of gradient damage and phase-field models for brittle fracture are compared.
- The phase-field model for brittle fracture can be conceived as a special gradient damage model.
- The right-hand side of the diffusion equation in the phase-field model introduces a length scale in the driving force.
- Together with the vanishing derivative of the degradation function this prevents a broadening of the damage zone.
- Numerical simulations corroborate the theoretical findings.

Abstract

Gradient-enhanced damage models and phase-field models are seemingly very disparate approaches to fracture. Whereas gradient-enhanced damage models find their roots in damage mechanics, which is a smeared approach from the onset, and gradients were added to restore well-posedness beyond a critical strain level, the phase-field approach to brittle fracture departs from a discontinuous description of failure, where the distribution function is regularised, leading to the inclusion of spatial gradients as well. Herein, we will consider both approaches, and discuss their similarities and differences. The averaging (diffusion) equations for the averaging field and the phase-field will be compared, and it is shown that the diffusion equation for the phase-field can be conceived as a special case of the averaging equation of a gradient-damage model where the damage is averaged. Further, the role of the driving force is examined, and it is shown that subtle differences in the degradation functions commonly adopted in damage and phase-field approaches are key to the observation that, different from damage mechanics, the fracture process zone does not broaden in the wake of the crack tip.

Keywords: Fracture; Damage; Gradient models; Phase-field approach

* Corresponding author.
E-mail address: r.deborst@sheffield.ac.uk (R. de Borst).

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1. Introduction

The numerical modelling to fracture can be approached from two different points of view. Discrete models for fracture, where the geometrical discontinuity is modelled as such, i.e. by modifying the geometry of the original, intact structure, are perhaps intuitively the most appealing approach to fracture, and have been pursued since the late 1960s [1]. Developments such as remeshing [2,3], or the eXtended Finite Element Method [4–8] have provided ways to decouple the crack path from the underlying discretisation. Also, isogeometric finite element analysis beholds promise to flexibly model propagating cracks [9].

Nevertheless, issues remain such as the proper modelling of curved crack fronts in three dimensions, while the robust implementation of discrete cracks in a three-dimensional setting is a non-trivial task, either when using remeshing, or when exploiting the partition of unity concept as in the eXtended Finite Element Method. Hence, smeared, or distributed, crack approaches have been put forward, where the discontinuity is distributed over a finite width. Another interpretation is that the Dirac function that arises for the strain at a discontinuity is replaced by a smooth function. The smearing out of the discontinuity is accompanied by the introduction, at local continuum level, of a stress–strain relation in which the limit strength is gradually reduced. The strain-softening that is introduced in this manner, however, locally leads to a change of the character of the governing partial differential equations: loss of ellipticity in case of quasi-static analyses, and loss of hyperbolicity for dynamic calculations.

This change causes a loss of well-posedness of the rate boundary value problem, which in turn causes a complete dependence of the numerical results on the discretisation, not only with respect to mesh refinement but also, and especially, with respect to mesh alignment, since failure zones exhibit a strong tendency to propagate along lines of discretisation. This tendency can be ameliorated by using elements in which the kinematics have been enriched by locally adding shape functions that can capture a discontinuity, e.g. [10–13]. However, to avoid loss of well-posedness, the standard, rate-independent continuum must be enhanced. Several possibilities exist: adding viscosity, e.g. [14,15], adding couple stresses and conjugate kinematic quantities like microcurvatures, micromorphic continua with the Cosserat continuum as the classical example [16], see [17,18] for a numerical implementation of Cosserat elasto-plasticity, spatial averaging [19], and the introduction of a dependence on spatial strain gradients, e.g. [20,21] for gradient plasticity, and [22–25] for gradient-enhanced damage models. Especially the latter class of models has become popular for computational analysis.

Another class of continuum descriptions of cracking has been developed in the context of brittle fracture. Pioneering work has been done by [26–28], who proposed a phase-field approximation of the variational formulation for Griffith's theory of brittle fracture based on the Mumford–Shah potential [29]. A more mechanically oriented formulation, which, however closely resembles the mentioned developments, has been derived by [30,31]. Subsequently, phase-field models have been applied to a large variety of fracture problems, including dynamic problems [32,33] and cohesive fracture [34].

However, the point of departure of both models is different. In gradient damage models intrinsically a mechanical approach is adopted, and the damage model is regularised by adding gradients to restore well-posedness of the boundary value problem in the post-peak regime. The basic idea of phase-field models, on the other hand, is to replace the zero-width discontinuity by a small, but finite zone with sharp gradients in a mathematically consistent manner. Indeed, the latter requirement inevitably leads to the inclusion of spatial derivatives in the energy functional, similar to gradient damage models.

To provide a proper setting we start by giving a brief outline of damage models, and their extension to nonlocality. An important issue in gradient damage models is the observation that in the wake of the crack tip there is a broadening of the damage field. To eliminate this broadening it has been proposed to make the internal length scale parameter a function of the local strain or damage level [35]. Next, a brief review of the phase-field approach to brittle fracture is given, and the different point of departure is emphasised. It is recalled that in this approach no broadening is observed of the damage zone. It is also argued that the regularisation parameter and the degradation function that are introduced are, in fact, material parameters, and have to be calibrated to experiments. A discussion on the differences and similarities between gradient-enhanced damage models and the phase-field approach to brittle fracture follows, including a comparison of the various formats of the diffusion equation for the damage/phase field that ensues for the different formulations and a discussion of the importance of the specific form of the driving force for the broadening of the damage zone.

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