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A discussion of fracture mechanisms in heterogeneous materials by means of configurational forces in a phase field fracture model

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Abstract

Composite materials are designed in order to combine the favorable properties of their components. For technical applications of these materials the determination of their effective material and fracture mechanical properties is of great interest. The study of fracture of heterogeneous composite materials, which is a necessary first step in order to derive effective fracture mechanical properties, is a difficult topic since macroscopic failure is usually accompanied by various fracture events on the microstructural scale. Finite element implementations of phase field fracture models enable the simulation of complex fracture scenarios as they occur in fracturing of heterogeneous materials. An adaption of the configurational force concept from linear elastic fracture mechanics to the phase field model provides more insight into the mechanisms underlying the evolution of fracture. In this work this ansatz is used to study the interplay between local crack driving forces and the evolution of fracture. Furthermore, the effects of the microstructure on macroscopically measurable quantities like the far field $\mathcal{J}$-integral are analyzed in representative examples comprising continuously modulated as well as layered structures with varying stiffness and varying cracking resistance.

Keywords: Phase field model; Fracture mechanics; Configurational forces; Heterogeneous material

1. Introduction

Fracture mechanical analysis of homogeneous materials is usually done by either using stress intensity factors (SIF) or energy release rates (ERR). The first concept is attributed to Irwin [1], while the second is associated with Griffith [2]. The SIF describe the intensity of the singular elastic fields at the crack tip to evaluate the criticality of a crack. The energetic approach (ERR) uses the idea that a crack grows if the mechanical energy provided by the deformation exceeds the energy required to create new fracture surfaces. Both of these criteria are merely designed to assess the criticality of a preexisting crack.

In heterogeneous materials the situation becomes much more complex as cracks experience crack arrest, repeated crack initiation, and crack deflection. An additional topic is the case of interface cracks between two materials, but

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this situation is beyond the scope of the present work. In order to model the discontinuous propagation of cracks in heterogeneous materials fracture models capable of describing crack nucleation, propagation, and complex paths are required. A model combining classical strength of materials concepts with the ERR concept is the finite fracture mechanics approach by Leguillon, e.g. [3–6]. It was shown to be able to analyze complex fracture scenarios. In addition size effects could be reproduced well by the model. However, it must be stated that it can be rather cumbersome to perform numerical simulations with the concept of finite fracture mechanics. This is due to the fact that different virtual crack lengths and directions need to be tested. The model uses two important material parameters, a rupture stress and a critical energy release rate. Other models, which have a similar conceptual idea, are phase field models. Often introduced in a way similar to non-local damage models these models describe the crack topology by introducing an additional order parameter. A rupture stress is implicitly incorporated via a length scale and the critical energy release rate. These models were first developed as a regularization of an energetic formulation of brittle fracture [7–12] and then extended in many directions including e.g. dynamic formulations [13–16], and inelasticity [17,18].

In recent investigations phase field models have been used to study heterogeneous materials [19]. Especially, the question of an effective fracture toughness of heterogeneous material is still an open question in the literature, although it is well known from many biological systems that a proper adjusted microstructure can increase macroscopic resistance to failure due to crack growth, see e.g. [20–22]. One of the main challenges in the discussion of the fracture toughness of heterogeneous materials originates from the fact that experimental observations can be difficult as the relation of macroscopic quantities to the crack driving force or $\mathcal{J}$-integral is not unique. Due to the material heterogeneity the $\mathcal{J}$-integral becomes path dependent and has a non trivial relation to the applied far fields, e.g. [23]. In principle the correction terms in the $\mathcal{J}$-integral can be computed, but they require the knowledge of the fields and the distribution of the heterogeneity, which is very cumbersome. In [23] it was shown that due to a harmonically distributed elastic stiffness the correlation between a far field $\mathcal{J}$-integral $\mathcal{J}^{\text{far}}$ and the crack driving force $\mathcal{J}^{\text{tip}}$ is very poor making the interpretation of experimental measurements ($\mathcal{J}^{\text{far}}$) very difficult.

The present investigation is along this line of discussion. A phase field model is used to model and simulate the complex crack propagation process. This acts as a virtual experiment. As will be demonstrated, different phenomena as crack arrest, crack re-initiation, crack re-nucleation at finite length, and stable crack growth occur depending on the heterogeneity of stiffness and fracture toughness. Smoothly varying properties as well as layered structures will be investigated. The interpretation of these phenomena will be done by configurational forces. In contrast the procedures in [24–27] the configurational force at the crack tip is not used to compute crack propagation, which simply follows from the time dependent Ginzburg–Landau equation of the fracture field (details are given in Section 2). The configurational force method is solely used to interpret the different driving and retarding mechanisms. In order to apply the theory to the phase field model the configurational force balance for a phase field continuum will be presented in Section 3. This approach is in line with the derivations presented in [11], but in the present work special attention is given to terms originating from the material heterogeneity. This can be understood as a generalization of the energetic approaches to fracture.

The main purpose of the simulations is to shed some light on how the generalized configurational forces relate to the local driving forces at the phase field crack and how these relate to $\mathcal{J}^{\text{far}}$ in different heterogeneous material scenarios. Besides this, scenarios are identified where other (stress based) approaches are needed to explain the evolution of fracture.

2. Phase field fracture model

The purpose of this section is to give a brief introduction into the phase field model for brittle fracture. The model follows ideas presented in [11] and is included here to make the present work self-contained. In order to describe the topology of cracks a fracture field s is introduced. The value $s = 1$ represents intact material, while $s = 0$ indicates cracks. The crucial part of the phase field model is the phase field potential, which depends on the strains $\boldsymbol{\varepsilon}$ and a fracture field s:

$$\Psi(\boldsymbol{\varepsilon}, s) = \int_{\Omega} \left[\underbrace{\left(s^2 + \eta\right) W(\boldsymbol{\varepsilon})}_{\psi^{\text{el}}(\boldsymbol{\varepsilon}, s)} + \underbrace{\mathcal{G}_c \left(\frac{1}{4\epsilon}(1 - s^2) + \epsilon |\nabla s|^2 \right)}_{\psi^{\text{fr}}(s)} \right] \mathrm{d}V, \quad (1)$$