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Fourth order phase-field model for local *max-ent* approximants applied to crack propagation

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Abstract

We apply a fourth order phase-field model for fracture based on local maximum entropy (LME) approximants. The higher order continuity of the meshfree LME approximants allows to directly solve the fourth order phase-field equations without splitting the fourth order differential equation into two second order differential equations. We will first show that the crack surface can be captured more accurately in the fourth order model. Furthermore, less nodes are needed for the fourth order model to resolve the crack path. Finally, we demonstrate the performance of the proposed meshfree fourth order phase-field formulation for 5 representative numerical examples. Computational results will be compared to analytical solutions within linear elastic fracture mechanics and experimental data for three-dimensional crack propagation.

Keywords: Fracture; Local maximum entropy; Second order phase-field model; Fourth order phase-field model

Abbreviations: LME, Local Maximum Entropy; *Max-ent*, Maximum entropy; FEM, Finite Element Method; GFEM, Generalized Finite Element Method; IGA, Isogeometric Analysis; XIGA, eXtended Isogeometric Analysis; XFEM, eXtended Finite Element Method; PDE, Partial Differential Equation; ODE, Ordinary Differential Equation; NURBS, Non-uniform Rational Basis Spline; l_0, Length scale parameter; h, Nodal spacing; β, Thermalization parameter to control the locality of the LME basis functions; γ, $\gamma = \beta h^2$, controls degree of locality; v, Phase-field variable; $\boldsymbol{u}$, Displacement tensor; $\boldsymbol{\varepsilon}$, Strain tensor; a, The crack length; G, Energy release rate; G_c, Critical energy release rate; F_0, The elastic energy density of an undamaged body; F, The elastic energy density of a body (damaged or undamaged); P, The external potential energy functional; E, The strain energy functional; Π, The total potential energy functional; Γ, The crack surface energy; Γ_{l_0}, The crack surface functional.

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1. Introduction

The modeling of fracture is of major importance in engineering applications such as aircraft fuselages, pressure vessels, automobile components, and castings. The progress in this field and the ability to prevent material failure have helped control the dangers caused by increasing technological complexity. A theoretical model for brittle fracture in solids was introduced by Griffith [1] and Irwin [2], which relates crack propagation to a critical value of the energy release rate. During the last few decades the numerical simulation of such process has gained importance and often plays a key role in design decisions [3–5]. This has been mainly motivated by the impossibility to have analytical solutions in most practical situations and the costs of obtaining meaningful and detailed information from experiments. Numerical methods, such as the finite element method have been used to model fracture with some success, but often they are unable to capture some physical properties of the phenomenon. Modeling of moving discontinuities with classical finite elements is difficult to automate because of the requirement that the mesh must conform to the surfaces of discontinuity. It also usually requires local refinement near the fracture zone, in particular near the crack tips where singularities in the stress field occur [6–9].

An attractive approach to overcome these difficulties has been presented by the extended finite element method (XFEM) [10,11] or the generalized finite element method (GFEM) [12]. These methods allow arbitrary propagating discontinuities without remeshing. One key challenge of such methods is describing the crack geometry and tracking the paths of the cracks as the fracture progresses. This becomes increasingly challenging for complex fracture patterns. In general these numerical approximations track the evolution of the fracture during the simulations but they have shown to be inefficient regarding, for example, crack branching in three dimensional applications. XIGA formulations (Extended isogeometric analysis) for fracture [13–15] aim to combine the advantages of isogeometric analysis and the extended finite element method. However, they also do not resolve the issue of complex track cracking procedures. An alternative to model complex fracture is meshfree methods [16–30]. While those contributions also rely on the representation of the crack surface, some meshfree methods such as the Cracking Particles Method [31,32] model fracture as a set of crack segments and therefore can capture – on cost of the accuracy in the crack kinematics – complex fracture patterns quite naturally.

Besides discrete crack models, continuous descriptions of fracture in solids have been presented. Among the most popular approaches are gradient models [33–38] and non-local models [39–42]. They introduce an intrinsic length scale and diffuse fracture over a certain width. Phase-field approaches for fracture [43] bear certain similarities to gradient models but they converge to a discrete crack model when the characteristic length tends to zero. Phase-field approaches also do not require an explicit representation of the crack surface and therefore complex crack tracking algorithms, see the recent contributions [44–49]. The crack surface is obtained as part of the solution and it is represented by an indicator function that is equal to 0 on the crack surface and 1 away from the fracture zone. The predecessors of phase-field approaches to fracture can be traced back to 1998 in [50,51], where the brittle crack propagation problem was regularized and recasted as a minimization problem. In this model, the proposed energy functional is closely similar to the potential functional presented by Mumford and Shah [52], which has been used in image segmentation. The existence of solution to the Mumford–Shah functional minimization was proven by Ambrosio in [53]. In [54], an approximation by an elliptic functional defined on Sobolev spaces was developed, based on the theory of Γ-convergence. In the phase-field approach, a continuous field governed by a partial differential equation (PDE) is used to model the cracks and their evolution. This method naturally deals with complex crack geometries. Its main drawback is the higher computational cost of solving a coupled PDE system.

Here we used phase-field model in combination with the local maximum-entropy (LME) method. The LME approximant schemes were developed in [55,56] using a framework similar to meshfree methods. The support of the basis functions is introduced as a thermalization (or penalty) parameter β in the constraint equations. In [55] it was shown that for some values of β, the approximation properties of the maximum-entropy basis functions are greatly superior to those of the finite element linear functions, even when the added computational cost due to larger support is taken into account. Subsequent studies [57–60], show that maximum entropy shape functions are suitable for solving a variety of problems such as linear and geometrically nonlinear thin shell analysis, compressible and nearly-incompressible elasticity and incompressible media problems. The LME approximations have several advantages over other meshfree methods such as the element-free Galerkin method [61] or Reproducing Kernel Particle Method as demonstrated also by numerical examples in [55,59,60,62,56]. For example, in contrast to above mentioned methods,

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