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Identifying the crack path for the phase field approach to fracture with non-maximum suppression

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Abstract

The phase field method is proven to be a powerful tool in computational fracture mechanics in that it can predict crack nucleation and branching, in some cases, without explicitly tracking the crack path. However, in such applications as hydraulic fracturing or leakage, it is necessary or beneficial to have the crack geometry available. This paper presents a variational method to identify the crack path from phase field approaches to fracture. The method is proven to be successful not only for a simple curved crack but also for multiple and branched cracks. The algorithm employs the non-maximum suppression technique, a procedure borrowed from the image processing field, to detect a bounding area which covers the ridge of the phase field profile. After that, it is continued with the step to determine a cubic spline to represent the crack path and to improve it via a constrained optimization process. To demonstrate the performance of our method, we provide the results with three sets of representative examples.

Keywords: Phase field; Non-maximum suppression; Fracture mechanics; Hydraulic fracturing; Erosion filter

1. Introduction

The phase field method [1–3] is a powerful way of simulating crack propagation in that it can predict crack emergence and branching, under certain circumstances, without explicitly tracking the crack path. One of the main ideas of this method is to employ a smeared representation of discrete cracks. However, in some applications it is still convenient to have the explicit crack path available, or even to develop a mechanism to introduce crack paths to

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partially replace a smeared crack propagation model, such as the approaches described in [4–6]. One application area is leakage problems [7].

Hydraulic fracturing is another example for which a crack identification scheme may be useful. This is a technique used in the oil and gas industry, where fractures are propagated by high-pressure liquid inside them, see [8] and references therein. With a phase field (or damage) approach, it becomes challenging how to impose the pressure loading on the rock mass exerted by the fracturing fluid, since most phase field approaches to date were developed for traction-free crack faces. Existing approaches that employ a phase field (or damage) approach to simulate hydraulic fracturing can be classified into two families.

The first family assumes either a known pressure field or a known displacement field [9–13]. Clearly an accurate simulation of the process requires considering a fully coupled system in order to obtain the spatial variation of the pressure field along the fracture, especially when the fluid viscosity is not negligible, as in most applications. Hence, this family of models can be considered as intermediate results towards more sophisticated models.

The second family defines an anisotropic permeability tensor [14,15] or an indicator function [16,17] related to the phase field, in order to obtain the fluid flow rate inside the fracture. When the solid of interest is at least moderately porous, these approaches are of technical importance; nevertheless, when the solid is barely permeable, having a pressure field defined everywhere seems costly as the phase field itself is already an additional scalar field to solve for, if one compares an explicit crack approach with the pressure field only defined on the crack path. In this case, having the crack path available from a phase field approach may offer a possibility of developing less expensive numerical schemes for impermeable solids.

Identifying the crack path from a phase field approach or damage model has been the interest of a couple of contributions. For example, Tamayo-Mas and Rodríguez-Ferran [18] proposed a medial-axis-based approach in the context of a continuous failure model characterized by a damage field. Therein, the authors proposed to model the crack path using the θ-simplified medial axis of a certain isoline of the damage variable. Bottoni et al. [19] developed another approach for searching the crack path from a damage field in up to three-dimensions. In the case of two-dimensions, they obtained location points one after another by finding the local maximum of the damage variable within a specific line at a distance from the last determined point.

In this work, we build a crack identification scheme by taking advantage of the variational structure of the phase field approach, although the scheme can also be applied to a solution from a damage model. The methodology is built on the so called *equivalent phase field* of a given curve, which is the generalization from the analytic phase field solution from the potential energy functional in a special case. The proposed algorithm is able to identify not only a simple crack but also a branched crack, as long as the crack path does not form a loop. The method is also capable of distinguishing multiple cracks close to each other up to a certain clearance.

This algorithm consists of three main steps. First of all, the non-maximum suppression method is employed to identify a bounding area within which the crack is believed to be. This is one of the main features of this paper, in that the gradient information of the phase field is exploited as well as the value. The bounding area provides the topological information of the crack path, and thus especially advantageous in the case of a branched crack. Then, a number of interpolation points are determined from this bounding area, from which a cubic spline is built as the initial guess for the crack. This step is comparable to the approach introduced in [19] but ours ensures: (1) a more uniform distance between the identified points and also (2) a stable weighted average expression to determine these points rather than local maximum. Afterwards, an optimization step is undertaken to improve the crack path. This is another feature of our paper, in that the crack tip and junction locations will be improved with this optimization step, even though the overall accuracy of the algorithm is limited by a resolution at best twice the mesh size. All of these steps are developed from the construction of the equivalent phase field.

The proposed algorithm is applicable to the phase field solution from mesh-based nonnegative basis functions, such as P_1, Q_1, and tensor product B-spline functions. In some sense, especially in the case of a Cartesian mesh of Q_1 elements, the first step of the method can be considered as an instance of an erosion filter. The use of a truncation method and non-maximum suppression corresponds to a classical filter and adaptive filter, respectively, the latter using the gradient information of the image – in our case the phase field – to construct structuring elements, see [20,21] for a related approach. Interested readers are referred to a broad review by Ćurić et al. [22].

The remaining of this paper will proceed as follows. In Section 2, we recall the phase field formulations for the variational description of brittle fracture, and introduce the concept of equivalent phase field of a given curve. Then in Section 3, we formulate the crack identification problem as an optimization problem by virtue of this equivalent phase

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