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A finite-strain solid–shell using local Löwdin frames and least-squares strains

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Highlights

- Finite strain solid–shell element with good distortion insensitivity.
- Shell benchmarking and comparison with established techniques.
- Motion of anisotropic axes by use of Lowdin frames.
- Constitutive framework based on a consistent updated-Lagrangian formulation with smoothed complementarity condition.
- Combination with standard 3D elements avoids additional tasks.

Abstract

A finite-strain solid–shell element is proposed. It is based on least-squares in-plane assumed strains, assumed natural transverse shear and normal strains. The singular value decomposition (SVD) is used to define local (integration-point) orthogonal frames-of-reference solely from the Jacobian matrix. The complete finite-strain formulation is derived and tested. Assumed strains obtained from least-squares fitting are *an alternative* to the enhanced-assumed-strain (EAS) formulations and, in contrast with these, the result is an element satisfying the Patch test. There are no additional degrees-of-freedom, as it is the case with the enhanced-assumed-strain case, even by means of static condensation. Least-squares fitting produces invariant finite strain elements which are shear-locking free and amenable to be incorporated in large-scale codes. With that goal, we use automatically generated code produced by AceGen and Mathematica. All benchmarks show excellent results, similar to the best available shell and hybrid solid elements with significantly lower computational cost.

Keywords: Finite-strain solid–shell; Least-squares; Assumed-strains; Singular value decomposition

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1. Introduction

Non-linear shell simulations with finite elements (cf. [1–3]) are peculiarly demanding with respect to numerical efficiency, Newton iteration robustness and mesh distortion insensitivity. This is relevant in the edge-based algorithms [4] when applied to quadrilaterals. Many of the intricate element formulations, such as enhanced-assumed-strain (EAS [5]), hybrid stress, discrete Kirchhoff (DK, cf. [6]), are suitable for smooth problems where the mesh distortion sensitivity is not a crucial ingredient and governing equations do not contain discontinuities. In addition, computational time associated with convergence difficulties and static condensation (specifically with EAS) can also be high. The same is true for meshless methods in shells (cf. [7,8]): geometrically complex problems with shells are difficult to solve with meshless methods. Beyond classical shell technology (cf. [9]), solid–shell elements such as [10,11] use continuum 3D constitutive laws without plane-stress enforcement and avoid rotational degrees-of-freedom. A pure displacement-based shell can be connected to standard continuum elements.

Nonlinear shell elements have been available for many decades and appeared to have reached the definite development with EAS (cf. [10]). However, recent availability of symbolic code generation [12] as an add-on to Mathematica has allowed more intricate formulations to be synthesized with moderate effort from the analyst. We take advantage of this to combine several established techniques and create a new solid–shell element. Specifically, we make use of the following techniques:

- Definition of equally-oriented local orthogonal frames by using the Jacobian matrix, singular value decomposition (SVD) and Löwdin algorithm [13].
- Least-squares in-plane assumed strains.
- Assumed natural transverse shear and normal strains.
- Constitutive framework based on a semi-implicit consistent updated-Lagrangian formulation.

A complete testing program is then performed. The set of obstacle problems for shells are the classical plate and shell benchmarks and extensions to finite strains. Besides thickness variation, it is important to test elements in finite strains since some instabilities have been found in the past (see [14] for a report with the Morley-based shell). In terms of low-order shell element technology, some important works should be mentioned. A milestone in the removal of transverse shear locking was achieved with the assumed natural strain (ANS) technique in 1984 and 1986 [9,15]. A decade earlier, in-plane bending locking for continuum elements was solved in 1973 by the Wilson Q6 element [16], with several ulterior corrections and applications to the in-plane part of shell strains. For undistorted meshes, convergence rate of the results is established regardless of the incomplete higher order terms in the polynomials (see the book by Belytschko and co-workers [17]) and these higher order terms contribute to stability and coarse-mesh accuracy. Of course, mesh distortion adversely affects the convergence rate (Lee and Bathe [18] proved the reduction of order of convergence) and it has been a problem with only a few published solution proposals, see also [19]. The advantage of using least-squares fitting for the in-plane strains is the excellent behavior with distorted meshes. This work is organized as follows: Section 2 presents the equilibrium equation for an arbitrary reference configuration, the full kinematics and the invariant frames. Section 3 describes the position vector and covariant metric components for a shell as well as the finite-strain algorithm. Section 4 introduces the specific discretization and assumed-strain components. The constitutive law is summarized in Section 5. Verification tests are shown in Section 6 and conclusions drawn in Section 7. Appendix describes the general consistent tangent modulus for any elasto-plastic model implemented with a semi-implicit integration method which.

2. Governing equations

2.1. Equilibrium for an arbitrary reference configuration

We obtain equations of equilibrium for an *arbitrary reference configuration* from the corresponding Cauchy equations of equilibrium (derivations for the latter are shown in Ogden [20]), which are given by:

$$\frac{\partial \sigma_{ij}}{\partial x_{a_j}} + b_i = 0 \tag{1}$$

with the Cauchy tensor components σ_{ij} $(i, j = 1, 2, 3)$. In (1) i is the direction index and j is the facet index. b_i are the components of the body force vector. Coordinates x_{a_j} are the spatial, or deformed, coordinates of a point belonging