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POD-Galerkin method for finite volume approximation of Navier–Stokes and RANS equations

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Highlights

- Reduced order modelling (ROM) for numerical simulation of fluid flows.
- Proper Orthogonal Decomposition (POD) with snapshots technique and Galerkin projection.
- Development of a ROM for CFD applications based on Finite Volume approximation.
- Employment of turbulent RANS simulations to extend the method to more industrial fields.
- Benchmark of the 2D lid-driven cavity both in laminar and turbulent conditions to assess the method.

Abstract

Numerical simulation of fluid flows requires important computational efforts but it is essential in engineering applications. Reduced Order Model (ROM) can be employed whenever fast simulations are required, or in general, whenever a trade-off between computational cost and solution accuracy is a preeminent issue as in process optimization and control. In this work, the efforts have been put to develop a ROM for Computational Fluid Dynamics (CFD) application based on Finite Volume approximation, starting from the results available in turbulent Reynold-Averaged Navier–Stokes simulations in order to enlarge the application field of Proper Orthogonal Decomposition-Reduced Order Model (POD-ROM) technique to more industrial fields. The approach is tested in the classic benchmark of the numerical simulation of the 2D lid-driven cavity. In particular, two simulations at $\mathrm{Re} = 10^3$ and $\mathrm{Re} = 10^5$ have been considered in order to assess both a laminar and a turbulent case. Some quantities have been compared with the Full Order Model in order to assess the performance of the proposed ROM procedure i.e., the kinetic energy of the system and the reconstructed quantities of interest (velocity, pressure and turbulent viscosity). In addition, for the laminar case, the comparison between the ROM steady-state solution and the data available in literature has been presented. The results have turned out to be very satisfactory both for the accuracy and the computational times. As a major outcome, the approach turns out not to be affected by the energy blow up issue characterizing the results obtained by classic turbulent POD-Galerkin methods.

Keywords: Proper orthogonal decomposition; Parametrized Navier–Stokes Equation; Reduced order modelling; RANS; Galerkin projection

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Nomenclature

a_i	time coefficient,
$\boldsymbol{a}$	ROM time coefficient vector,
$\boldsymbol{B}$	ROM matrix,
$\boldsymbol{BT}$	ROM matrix,
$\boldsymbol{C}$	ROM matrix,
$\boldsymbol{CT1}$	ROM matrix,
$\boldsymbol{CT2}$	ROM matrix,
$\boldsymbol{D}$	ROM matrix,
e	L^2 error of velocity,
$\boldsymbol{E}$	ROM matrix,
F	face flux, m^3 s^{-1}
F_r	ROM face flux, m^3 s^{-1}
k	turbulent kinetic energy, m^2 s^{-2}
$\boldsymbol{n}$	normal vector,
N_r	number of ROM functions,
N_s	number of snapshots,
p	normalized pressure, m^2 s^{-2}
p_r	ROM normalized pressure, m^2 s^{-2}
t	time, s
$\boldsymbol{u}$	velocity, m s^{-1}
$\boldsymbol{u}_n$	snapshots velocity, m s^{-1}
$\boldsymbol{u}_r$	ROM velocity, m s^{-1}
$\boldsymbol{u}_t$	derivative of the velocity, m s^{-1}
$\boldsymbol{u}_{BC}$	Dirichlet boundary condition of velocity, m s^{-1}
$\boldsymbol{u}_{FOM}$	FOM velocity, m s^{-1}
$\boldsymbol{x}$	vector of spatial coordinate, m
V	volume, m^3

Greek symbols

Γ	boundary function,
υ	kinematic viscosity, m^2 s^{-1}
υ_{EV}	eddy viscosity in POD-G-ROM, m^2 s^{-1}
υ_t	turbulent viscosity, m^2 s^{-1}
$\upsilon_{t,r}$	ROM turbulent viscosity, m^2 s^{-1}
τ	penalty factor,
$\boldsymbol{\varphi}_i$	velocity spatial modes, m s^{-1}
ϕ_i	turbulent viscosity spatial modes, m^2 s^{-2}
χ_i	pressure spatial modes, m^2 s^{-2}
ψ_i	face flux spatial modes, m^3 s^{-1}
ω	specific dissipation, s^{-1}
Ω	spatial domain, m^3

1. Introduction

Numerical simulation of fluid flows requires a strong computational effort but it is essential in engineering applications. Even if the computational power is becoming more and more available, the need of finding a trade-off between computational cost and solution accuracy is a preeminent issue especially in process optimization, control or in general in any real time or many query contexts [1–5]. A viable solution is to employ Reduce Order Modelling (ROM) techniques [6]. The aim of a computational reduction technique is to retain the governing dynamics of a