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ScienceDirect
Computer methods in applied mechanics and engineering
Comput. Methods Appl. Mech. Engrg. 311 (2016) 180–207
www.elsevier.com/locate/cma

Convergence analysis of multirate fixed-stress split iterative schemes for coupling flow with geomechanics

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Received 6 March 2016; received in revised form 22 July 2016; accepted 25 July 2016
Available online 3 August 2016

Abstract

We consider multirate iterative schemes for the Biot system modeling coupled flow and geomechanics in a poro-elastic medium. The multirate iterative coupling scheme exploits the different time scales for the mechanics and flow problems by taking multiple finer time steps for flow within one coarse mechanics time step. We adapt the fixed stress split algorithm that decouples the flow and mechanics equations for the multirate case and perform an iteration between the two problems until convergence. We provide a fully discrete scheme that uses Backward Euler time discretization and mixed spaces for flow and conformal Galerkin for mechanics. Our analysis is based on studying the equations satisfied by the difference of iterates and using Banach contraction argument to prove that the corresponding scheme is a fixed point contraction. The analysis provides the value of an adjustable coefficient used in the proposed iterative coupling algorithms. Furthermore, we show that the converged quantities satisfy the variational weak form for the coupled discrete system.

Keywords: Poroelasticity; Biot system; Fixed-stress split iterative coupling; Multirate scheme; Contraction mapping

1. Introduction

The accurate modeling of coupled fluid flow and mechanical interactions has received more attention and importance for both environmental and petroleum engineering applications. Accurate and reliable numerical methods for solving such problems are needed for the accurate modeling of multiscale and multiphysics phenomena such as reservoir deformation, surface subsidence, well stability, sand production, waste deposition, pore collapse, fault activation, hydraulic fracturing, CO_2 sequestration, and hydrocarbon recovery [1,2]. Traditionally, changes in mechanical deformations are visible to fluid flow through a pore compressibility factor, which is insufficient for stress sensitive, and structurally weak reservoirs. In fact, it is only through the coupling between flow and mechanics that

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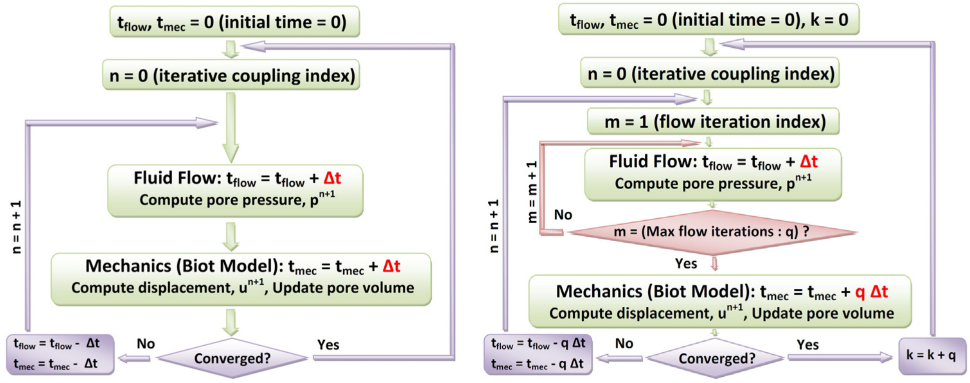
http://dx.doi.org/10.1016/j.cma.2016.07.036
0045-7825/



(a) Single rate. (b) Multirate.

Fig. 1.1. Flowchart for iterative algorithm using single rate and multirate time stepping for coupled geomechanics and flow problem.

reliable reservoir models can be obtained. In several of the applications listed above, the mechanics and flow equations have different characteristic time scales. Multirate schemes exploit the different time scales of these two equations and allow taking different time steps for each of these two problems. This is naturally achieved by decoupling the two equations.

There are typically three different coupling approaches employed in modeling fluid flow coupled with reservoir geomechanics. They are known as the fully implicit, the explicit or loose coupling, and the iterative coupling methods. The fully implicit approach solves reservoir multiphase flow and mechanics equations simultaneously and enjoys excellent stability properties [3] though it poses certain computational challenges for the linear solver. On the other hand, the loosely coupled approach is less accurate, only conditionally stable but, contrary to the implicit coupling scheme, has a lower computational cost. The iterative coupling approach lies in between the two extremes, and solves the two coupled subsystems iteratively by exchanging the values of the shared state variables in an iterative manner. The procedure is iterated at each time step until the solution is obtained with an acceptable tolerance [2,4–6]. The iterative coupling approach allows the use of existing reservoir simulators, is easy to implement, is robust and has fast convergence provided it has been designed appropriately. Our proposed numerical method is based on such an iterative approach. These iterative methods can be also used as a pre-conditioner for the fully implicit method. The work of Gai et al. [7,8] interpreted the fixed stress split iterative coupling scheme as an effective physics-based preconditioning strategy applied to a Richardson fixed point iteration. The same preconditioning operator can be applied to the fully implicit coupled system, enhancing the underlying Krylov subspace iteration as well [7,9,10].

The coupled flow and geomechanics problem has been intensively investigated in the past. Starting from the pioneering work of Terzaghi [11] and Biot [12,13], several nonlinear extensions have been proposed and investigated [14–16]. The work of Settari and Mourits [17] proposed a robust iterative and explicit coupling schemes for coupling flow with geomechanics along with fracture propagation. The existence, uniqueness, and regularity of the Biot system have been investigated by a number of authors (Showalter [18], Phillips & Wheeler [19], and Girault et al. [20]). However, the development and analysis of theoretically convergent iterative coupling algorithms in poro-elastic media have received relatively less attention. Recently, the work of Mikelić and Wheeler [21] establishes geometric convergence (contraction with respect to appropriately chosen metrics) for different flow and geomechanics iterative coupling schemes. In addition, Kim et al. [3,22] have used von Neumann stability analysis to study the stability and convergence of similar schemes. Moreover, the stability of the multirate explicit coupling schemes has been recently investigated and established for the quasi-static Biot system under mild conditions on material parameters [23].

Our work is inspired from the previous work of Mikelić and Wheeler [21] (see also [24]) and extends their results to cover the case of fully discrete multirate iterative coupling schemes. Convergence properties of multirate explicit coupling schemes have been heavily investigated in [25,26] for the non-stationary Stokes–Darcy model. In contrast, we consider multirate iteratively coupled flow and geomechanics problems in this work. Fig. 1.1(a) and 1.1(b) illustrate the differences between single rate versus multirate iterative coupling schemes. Fig. 1.1(a) represents a typical single rate scheme, in which the flow and mechanics problems share the exact same time step, and the coupling iteration continues until convergence. In contrast, Fig. 1.1(b) demonstrates a typical multirate scheme, in