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Implicit finite incompressible elastodynamics with linear finite elements: A stabilized method in rate form.

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Abstract

We propose a stabilization method for linear tetrahedral finite elements, suitable for the implicit time integration of the equations of nearly and fully incompressible nonlinear elastodynamics. In particular, we derive and discuss a generalized framework for stabilization and implicit time integration that can comprehensively be applied to the class of all isotropic hyperelastic models. In this sense the presented development can be considered an important extension and complement to the stabilization approach proposed by the authors in previous work, which was instead focused on explicit time integration and simple neo-Hookean models for nearly-incompressible elasticity. With the goal of computational efficiency, we also present a two-step block Gauss-Seidel strategy for the time update of displacements, velocities and pressures. Specifically, a mixed system of equations for the velocity and pressure is updated implicitly in a first stage, and the displacements are updated explicitly in a second stage. The proposed mixed formulation is then embedded in Newton-type strategies for the nonlinear solution of the equations of motion. Various implicit time integration strategies are considered, and, particularly, we focus on high-frequency dissipation time integrators, which are preferable in transient mechanics applications. An extensive set of numerical computations with linear tetrahedral elements is presented to demonstrate the performance of the proposed approach.

Key words: Tetrahedral finite element; piece-wise linear interpolation; stabilized method; transient dynamics; incompressible elasticity.

1. Introduction

Many applications in computational mechanics involve incompressible materials. For example, the behavior of biomaterials, which are predominately made of aqueous solutions, is often approximated as incompressible [11, 72, 114]. Additional examples are found in polymeric compounds (including most rubbers), and certain types of soils and geologic strata.

The constraint of incompressibility is difficult to treat numerically as it gives rise to saddle-point problems and associated Ladyzhenskaya-Babuška-Brezzi (LBB) conditions [13, 22, 46] (also known as *inf-sup* conditions). Spurious pressure checkerboard modes and volumetric locking [76] are instantiations of the pathologies associated with finite elements that do not satisfy the LBB conditions. An additional element of complexity considered in this work is the simulation of time-dependent problems, which may pose specific challenges to methods initially developed for static computations.

In most engineering applications, the design geometry may be quite complex [27, 102], and this aspect poses important restrictions on the choice of numerical methods. To date, automatic meshing in complex geometry is confined to simplicial (i.e., tetrahedral and triangular) grids. In fact, Delaunay and other advancing front algorithms have proved a mature, robust, and widespread technology for simplicial mesh generation [35, 58, 95, 135]. However, a problematic issue is that there are no simple tetrahedral finite element formulations that are robust and accurate, particularly in the time-dependent, nearly or fully incompressible cases. Current available methods typically involve rather complicated implementations, and, in some cases, they lack stability/robustness. On the other hand, a number

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