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Abstract

In this paper we present an application of the reduced basis method to a local high definition adjusted wind model. The model provides a precise description of the wind in 3D and takes into account topography and thermal gradients on the surface by solving only 2D linear equations; the buoyancy forces, slope effects, and mass conservation are also considered. The wind field is adjusted to the point measurements through an optimal control problem in which the wind flux acts as a control on the boundary. In order to use a reduced basis method, we consider an affine decomposition in terms of the parameter related to the friction coefficient and the wind measures at some given observation points. We also design an a posteriori error estimator that is needed to conduct our reduced basis process. Finally, two numerical examples are presented: a test problem and a real-data scenario, we corroborate the correct behavior of the method in both cases.

Keywords: Reduced basis method, wind field adjustment, optimal control, A posteriori error estimates

1. Introduction

In this paper we implement the reduced basis method for the efficient resolution of an optimal control problem associated to a wind field model. Wind models are a fundamental tool in the study of environmental problems such as dispersion of pollutants, fire propagation, among others. Our starting point is a mass consistent vertical diffusion wind field model. If the significant phenomena that we want to simulate occur in a zone, where the horizontal dimensions are much larger than the vertical one, then an asymptotic approximation of the primitive Navier–Stokes equations can be derived as in the model developed in [1, 2]. The most salient feature of this asymptotic approach is that it provides a three-dimensional velocity wind field (which satisfies the incompressibility condition in the air layer) governed by a two-dimensional equation, so that it can be coupled with the temperature surface distribution in order to take into account the thermal effects such as sea breezes. In addition, the terrain elevation information is also taken into account by the model.

The validity of this model has the following limits: the nonlinear terms are neglected and it is assumed that the air temperature decreases linearly with the height. On the other hand, the model takes into account buoyancy forces, slope effects, and mass conservation. The wind model presented in this paper is an adaptation of the wind model proposed in [1]. When the data are given by meteorological predictions, an optimal control problem is obtained [2], which can be solved using the adjoint equation-based method. We refer the reader to [3] for the details of this approach. The corresponding numerical approximation leads to linear algebraic systems of equations that are very ill conditioned and quite challenging to solve. In practical applications, the number of equations can be high (roughly between 100 000 and 600 000) and the algebraic system has to be solved many times in the course of a simulation. Moreover, the model requires the estimation of the involved parameters. This adjustment is usually made by genetic algorithm [4], and demands intensive computation. Therefore, the search for efficient solver methods is needed.

The reduced basis method is a procedure for the efficient solution of parametrized partial differential equations. A general formulation of the method and an analysis of their properties can be found in [5] (see also [6]). This method is premised upon a *trustworthy* technique to approximate the original problem, in our case the finite element