Accepted Manuscript

Implicit Hybrid Upwind scheme for coupled multiphase flow and transport with buoyancy

François P. Hamon, Bradley T. Mallison, Hamdi A. Tchelepi

PII: S0045-7825(16)30897-0
DOI: http://dx.doi.org/10.1016/j.cma.2016.08.009
Reference: CMA 11090

To appear in: *Comput. Methods Appl. Mech. Engrg.*

Received date: 27 March 2016
Revised date: 8 August 2016
Accepted date: 9 August 2016

Please cite this article as: F.P. Hamon, B.T. Mallison, H.A. Tchelepi, Implicit Hybrid Upwind scheme for coupled multiphase flow and transport with buoyancy, *Comput. Methods Appl. Mech. Engrg.* (2016), http://dx.doi.org/10.1016/j.cma.2016.08.009

Implicit Hybrid Upwind Scheme for Coupled Multiphase Flow and Transport with Buoyancy

François P. Hamon[a,*], Bradley T. Mallison[b], Hamdi A. Tchelepi[a,]

[a]*Energy Resources Engineering, Stanford University, California, USA*
[b]*Chevron ETC, 6001 Bollinger Canyon Rd, San Ramon, California, USA*

Abstract

Numerical simulation of coupled multiphase flow and transport in porous media is used in many scientific and industrial applications, such as groundwater management and oil recovery. To solve the governing partial differential equations, we focus on the fully-implicit (backward-Euler) finite-volume method (FIM). In this numerical scheme, large algebraic systems must be solved at each time step using Newton's method. This can be quite expensive, especially for highly nonlinear problems with buoyancy. In this work, we present a new first-order approximation of the flux in the FIM leading to improved nonlinear convergence rate and radius. This approximation is based on Implicit Hybrid Upwinding and relies on a separate treatment of the viscous and buoyancy parts of the numerical flux. Moreover, in the viscous part, the total velocity discretization is key, as it controls the coupling between the elliptic flow problem and the hyperbolic multiphase transport problem. To achieve robustness and efficiency, we propose a differentiable total velocity discretization which adapts to the balance of forces at each interface. We analyze the theoretical properties of the resulting numerical scheme and prove that saturations remain between physical bounds. Challenging two- and three-phase test cases illustrate that our numerical scheme brings a significant reduction in the number of nonlinear iterations compared to a finite-volume discretization relying on the widely used phase-per-phase upstream weighting. Our numerical scheme therefore reduces the simulation cost while providing similar accuracy.

Keywords: Porous media, Flow and transport, Three-phase flow, Implicit finite-volume schemes

1. Introduction

Numerical simulation of subsurface fluid flow is key to better understand underground hydrosystems and oil recovery processes. It involves solving the partial differential equations (PDEs) governing coupled multiphase flow and transport in porous media. In these equations, strong heterogeneities in porosity and permeability lead to differences of several orders of magnitude in the transport time scales. Solution methods based on explicit time discretization suffer from a severe restriction on the time step size needed to honor the stability condition. For this reason, a fully-implicit (backward-Euler) time discretization offering unconditional stability is often preferred. However, solving the algebraic systems arising from a fully-implicit finite-volume discretization of the PDEs can be quite challenging for nonlinear solvers. The elliptic flow and the highly nonlinear hyperbolic transport of species cannot be decoupled (Aziz and Settari (1979); Peaceman (2000)). Therefore, in the solution method, large and ill-conditioned algebraic systems must be solved at each time step using a nonlinear solver – often Newton's method with damping (Deuflhard (2011)). Thus, enlarging the nonlinear convergence radius and improving the convergence rate are key to reduce the computational cost of a simulation, and enable large-scale problems to be solved in practice.

In subsurface applications, Phase-Potential Upwinding (PPU) is a commonly used approximation of the numerical flux in the finite-volume method. In PPU, the mobility of each phase is upwinded separately based on the sign of the phase potential difference at the interface between two control volumes (Brenier

*Corresponding author